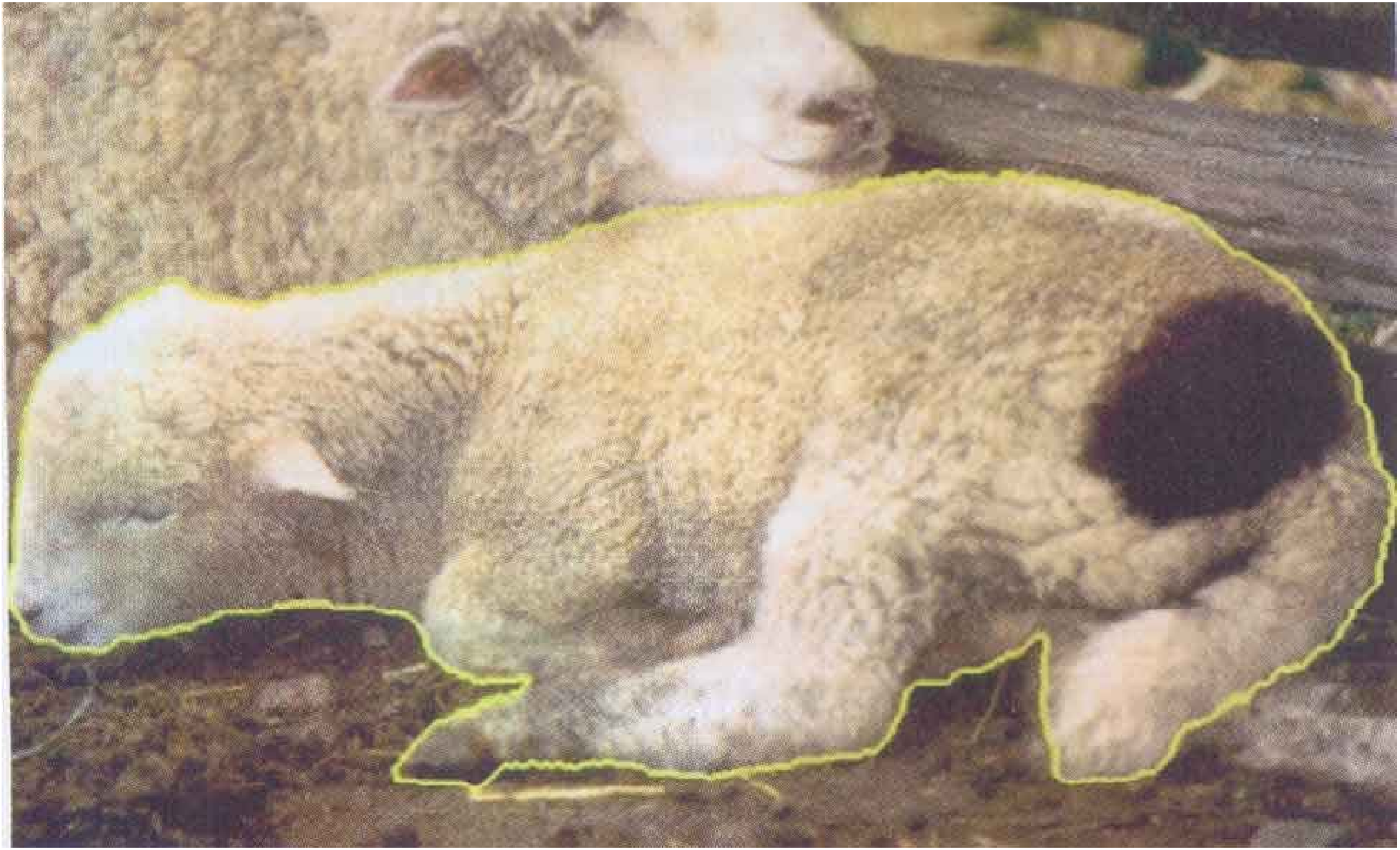
Image Segmentation and convex hull compuation

Artificial picture constructed from the segmented images



A popular automated system: SNAKES

Input initial boundary curve (rubber-band)

The rubber-band shrinks to capture the region boundary

Question: Can we solve the problem as a simple mathematical problem?

History

- Goes back to 1994 (15 years ago)
- Tetsuo Asano, Naoki Katoh, and I tried to formulate and solve the **image segmentation** problem as a geometric optimization problem
- Surprisingly, convex hull plays an important role.

Image segmentation problem

- **G** = n x n pixel grid (for example, n =1024)
- A digital picture is a function **f (x)** on G to represent brightness/color of each pixel x
  - f(x) is real valued (monochromatic picture)
    - In RGB space for color pictures
- **Object image** is a subset S of G to represent an object in the picture.
- Image segmentation: Clip the object image

Our formulation

- Approximation by two-valued function
  - Picture: function f from G to real values
  - Find the L_2 nearest two-valued function g to f

$$g(p) = a \quad (p \in R)$$ Image

$$g(p) = b \quad (p \notin R)$$ background

Minimize $\| f - g \|_2$

- a and b become average values μ(R) and μ (G-R) of f(p) in R and G-R, respectively.

- That is, minimize the intraclass variance

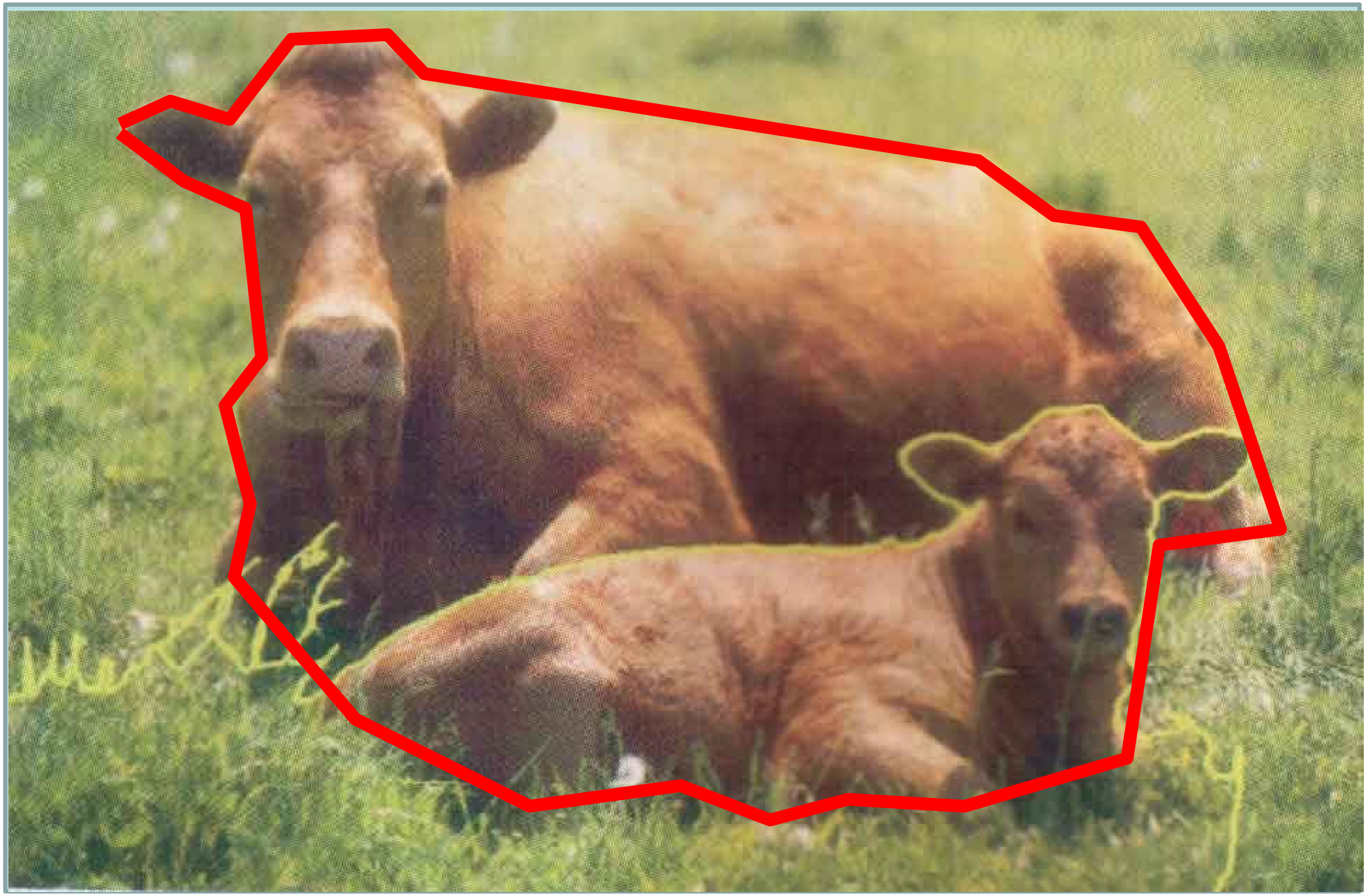
$$Var(R) = \sum_{p \in R} (f(p) - \mu(R))^2 + \sum_{p \in G-R} (f(p) - \mu(G-R))^2$$

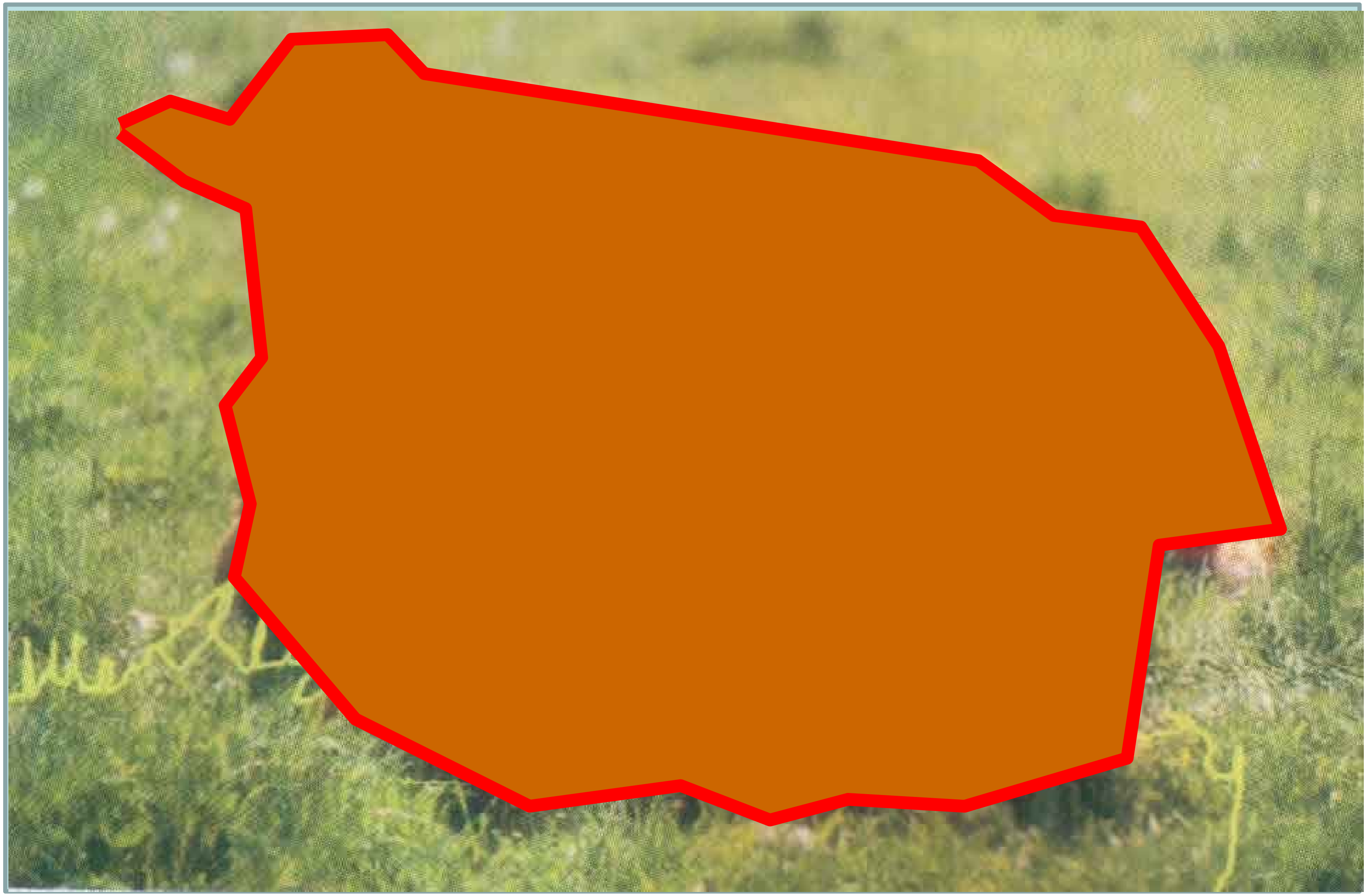
Intraclass variance minimization

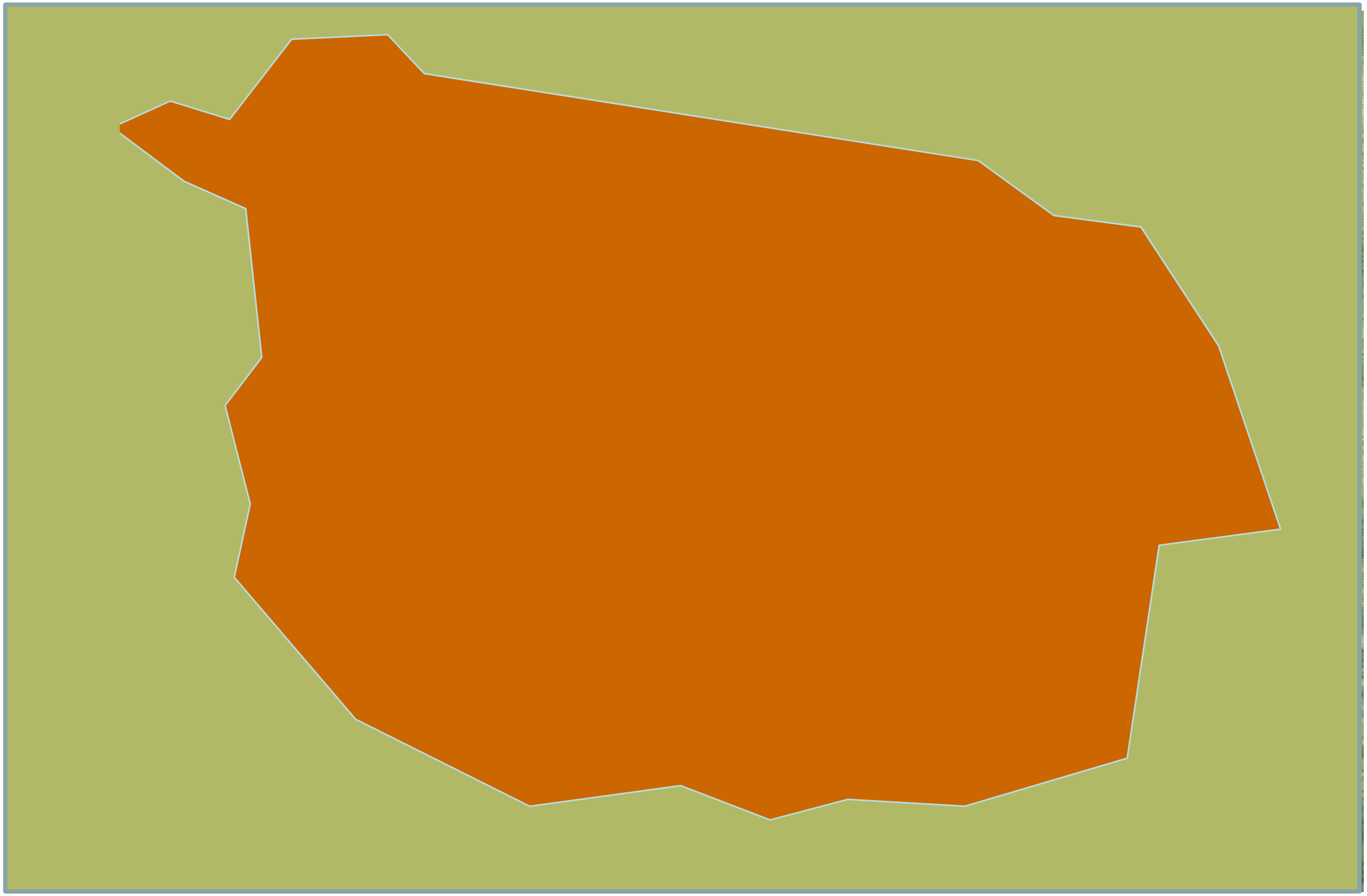
$$Var(R) = \sum_{x \in R} (f(x) - \mu(R))^2 + \sum_{x \in G-R} (f(x) - \mu(G-R))^2$$

- Easy if R can be arbitrary (disconnected) region.
  - Least-square threshold selection (Ohtsu, 1978)
    - Collect pixels brighter than a threshold θ

- Reasonable formulation: Give a family **F** of regions of good shapes, and find R ∈ **F** minimizing Var (R)

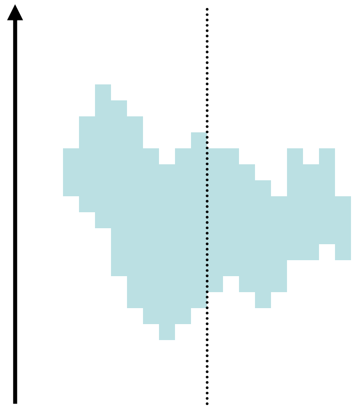




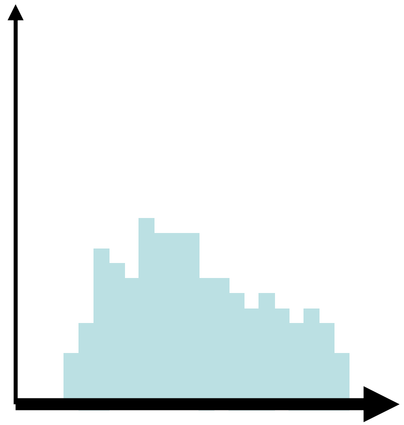


Typical Region Families

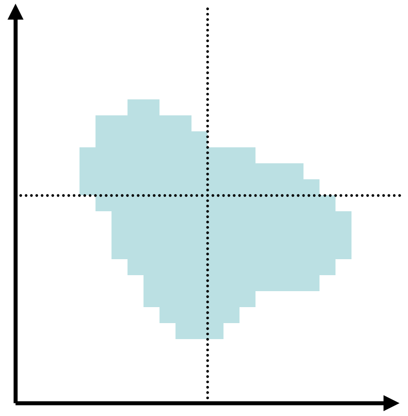
X-monotone



Based x monotone



Rectilinear Convex



X-monotone: Intersection with any vertical line is a segment. (bounded by two x-monotone chains)

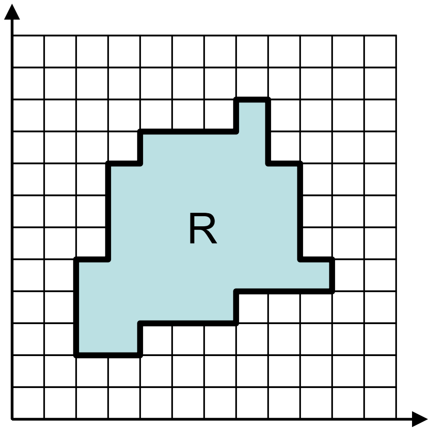
Based (x-)monotone: Region bounded by a monotone chain and a baseline (x-axis)

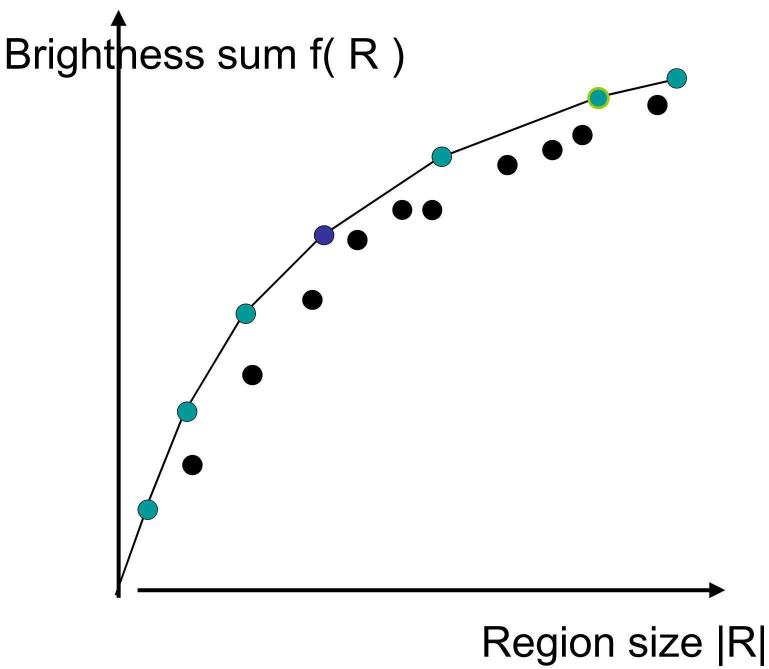
Rectilinear Convex: X-monotone and Y-monotone region.

Solution (Asano-Chen-Katoh-T 96)

- Idea :
  - If we fix the number k= |R| of pixels in R, Var(R) is minimized if the sum $f(R) = \sum_{p \in R} f(p)$ is maximized (or minimized).
    - To compute such R is NP-hard even for the base monotone regions
  - Because Var (R) has convexity, we can use convex hull computation to solve it.

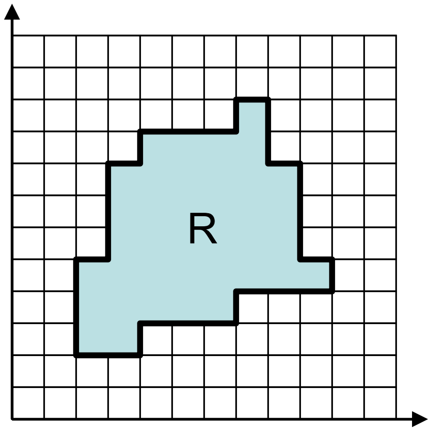
Convex hull computation



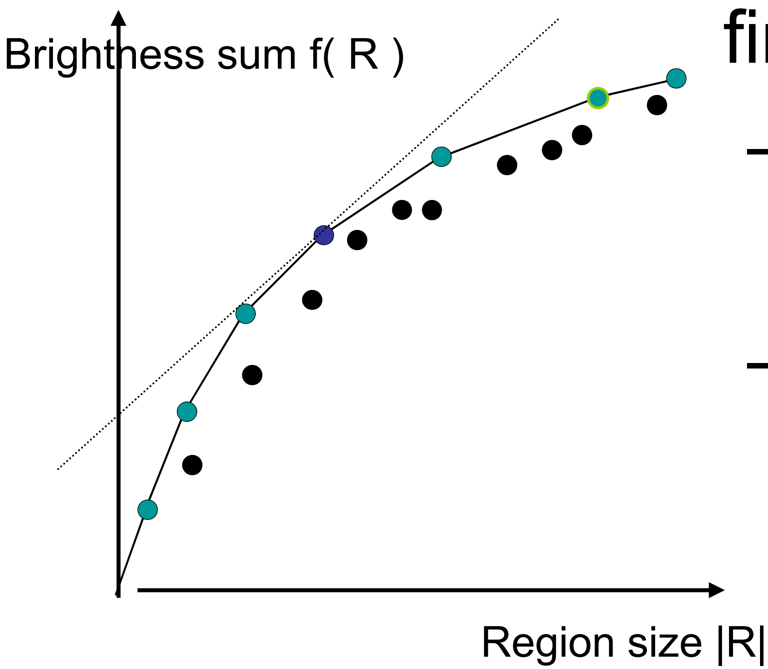


- Consider S = {(|R|, f(R)): R ∈**F**}.
- **F** has an exponential number of regions in general
- Thus, we cannot compute S
- Fortunately, CH(S) has at most $N=n^2$ points
  - Output size is small.
- Var(R) is maximized at a point on CH(S) (by its convexity)
- Problem: How can we compute CH(S) without knowing S explicitly

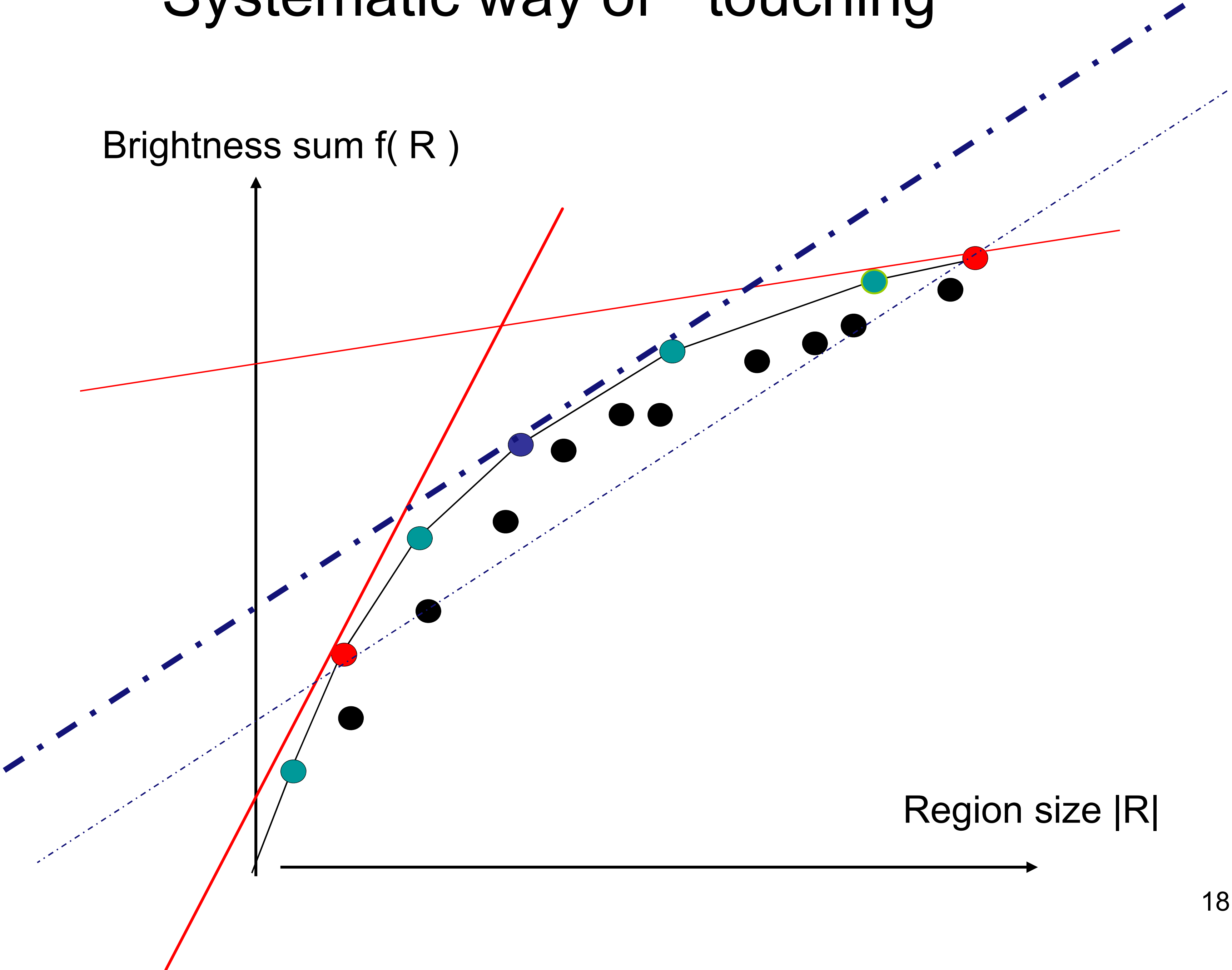
Convex hull computation by “touching” it



- Compute CH(S) by finding all the tangent points
  - Recall points are (|R|, f(R))
  - Tangent line: $y - t\,x = c$
- **Maximizing f*(R)** = f (R)– ***t*** |R| finds a tangent point with slope ***t***
  - $f^*(R) = \Sigma_{p \in R} f^*(p)$, where $f^*(p) = f(p) - t$
  - We can find all necessary slopes by touching O(N) times. (works even in high dimensions)



Systematic way of “touching”



The problem we need to solve

Maximum weight region problem:
Given a function f*(p) on G, find the region R in the region family **F** maximizing f* (R)

History: Programming Pearls (1984), column 8
(J. Bentley's famous column CACM)

- *How to solve it if F is*
  - *the family of all rectangles*
  - *the family of all intervals in the one-dimensional array*

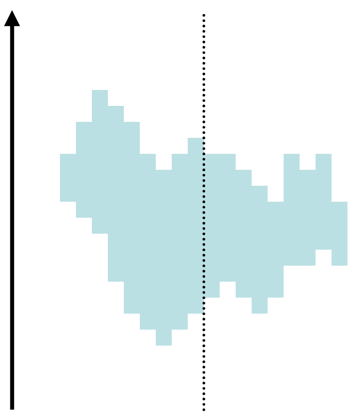
Maximum weight region

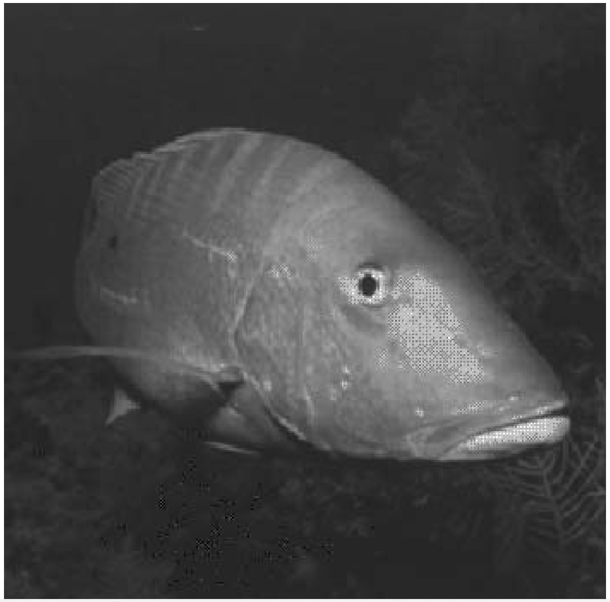
Maximum weight region problem:
Given a function f*(p) on G, find the region R in the region family **F** maximizing f* (R)

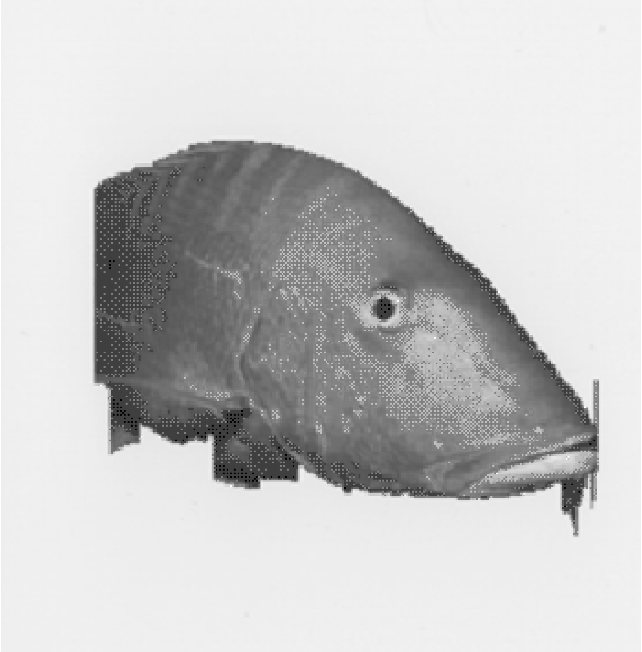
Easy to solve if **F** is the family of

- Based x-monotone regions
- (Connected) x-monotone regions
- Rectilinear convex regions

NP-hard for the family of all connected regions








I was lucky to find unexpected applications and extensions

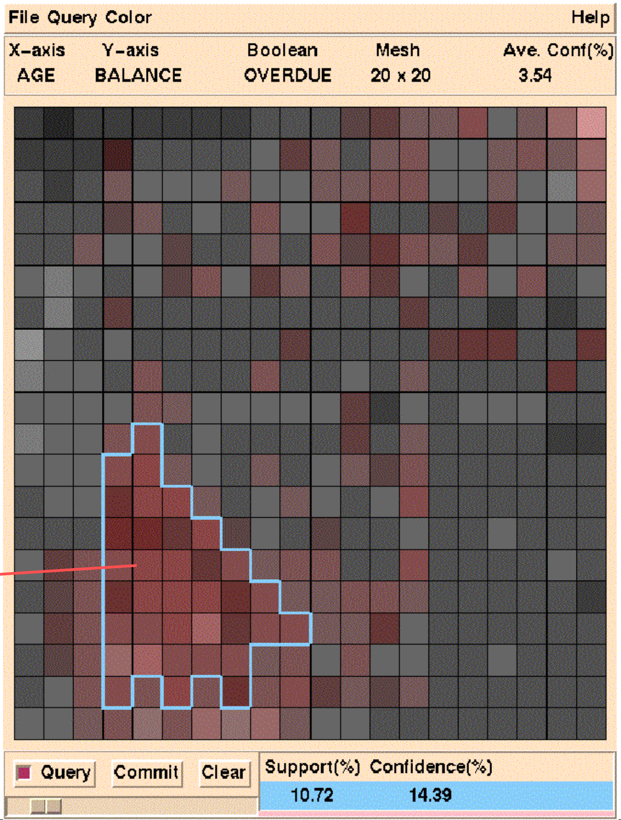
•Data Mining Application: Optimized Numeric Association Rules (SIGMOD 96 ,VLD96,98, KDD 97)

SONAR

(System for Optimized Numeric Association Rules)

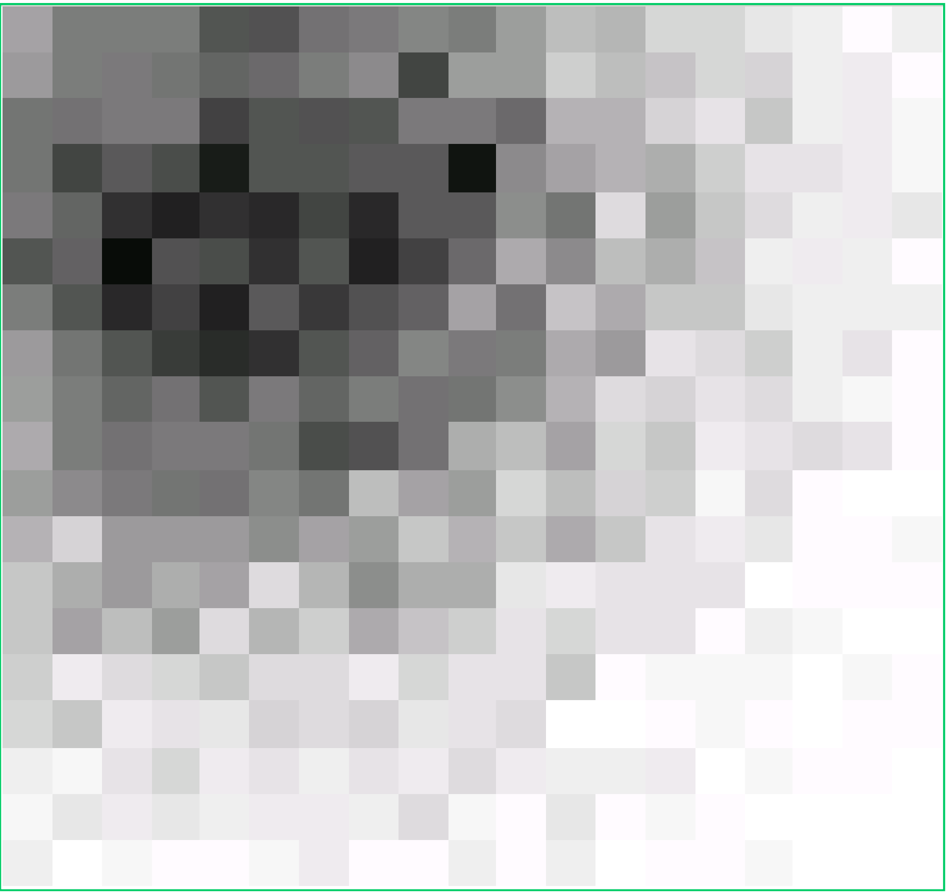
Find a rule to detect unreliable customers using a customer database

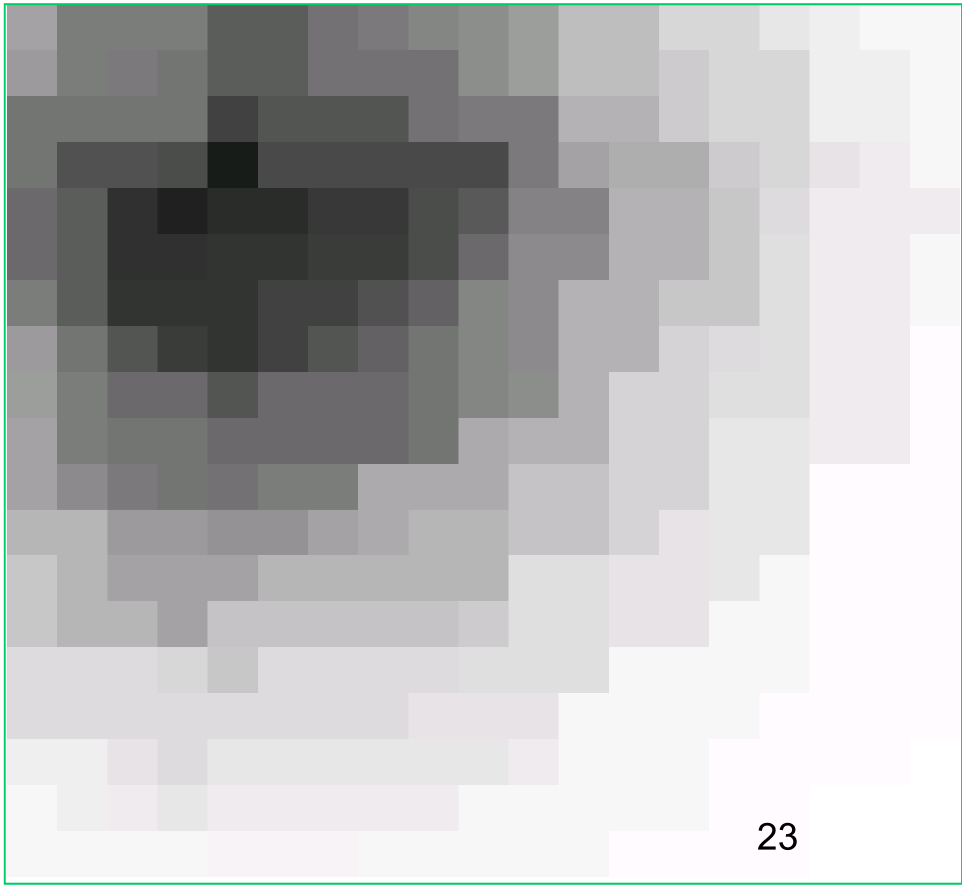
(Age, Balance) $\in$ R
$\Rightarrow$ (CardLoanDelay = yes)



Pyramid approximation and layered rule (Chun-Sadakane-T 03, Chen-Chun-Katoho-T 04)

Instead of two-valued function, we can construct the optimal multilayer function to approximate the input f.





Remained problems

- The region families are very special
- How to deal with more flexible regions
  - A region consisting of a few basic shapes
  - Convex region, Star-shaped region