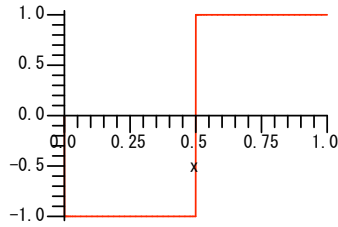


```

> restart;
> #F:=x->piecewise(x=0,1/2,x>0,x);
#F:=x->piecewise(x<1/2,x,x>=1/2,1-x);
#F:=x->piecewise(x<1/2,-1,x>1/2,1);
#F:=x->piecewise(x<1/2,-1,x>=1/2,1);
F:=x->piecewise(x>0 and x<1/2,-1,x>1/2,1);
plot(F(x),x=0..1);

F:=x->piecewise(0<x and x<1/2,-1,1/2<x,1)

```



```

> KK:=2;
N:=2^KK;L:=1-0;

KK:=2
N:=4
L:=1

```

Direct Integrations for Fourier Series

```

> for n from 0 to N do
a[n]:=2/L*int(F(x)*cos(2*Pi*n/L*x),x=0..L);
end do;

a0:=0
a1:=0
a2:=0
a3:=0
a4:=0

> for n from 0 to N do
b[n]:=2/L*int(F(x)*sin(2*Pi*n/L*x),x=0..L);
end do;

b0:=0
b1:=-4/pi
b2:=0

```

$$b_3 := -\frac{4}{3\pi}$$

$$b_4 := 0$$

```

> for n from 0 to N do
c[n]:=1/L*int(F(x)*exp(-I*2*Pi*n/L*x),x=0..L);
end do;
for n from 1 to N do
c[-n]:=1/L*int(F(x)*exp(I*2*Pi*n/L*x),x=0..L);
end do;

```

Note that:
 $a[n]=c[n]+c[-n]$, $b[n]=I(c[n]-c[-n])$
 $c[-n]=1/2(a[n]-I b[n])$

$$c_0 := 0$$

$$c_1 := \frac{2I}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2}{3} \frac{I}{\pi}$$

$$c_4 := 0$$

$$c_{-1} := -\frac{2I}{\pi}$$

$$c_{-2} := 0$$

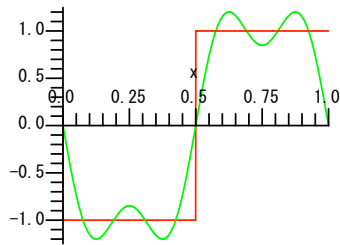
$$c_{-3} := -\frac{2}{3} \frac{I}{\pi}$$

$$c_{-4} := 0$$

```

> F1:=unapply(sum(evalf(c[i]*exp(I*2*Pi*i/L*x)),i=-(N-1)..(N-1)),x);
> plot({Re(F1(x)),F(x)},x=0..1);

```



Orthogonality for the integration

```
> for k from 0 to N-1 do
  c[k]:=evalf(sum(F(i*L/N)*exp(-I*2*Pi*k*i/N),i=0..N-1));
end do;
```

$c_0 := 0.$

$c_1 := 4.828427124 I$

$c_2 := 0.$

$c_3 := 0.828427124 I$

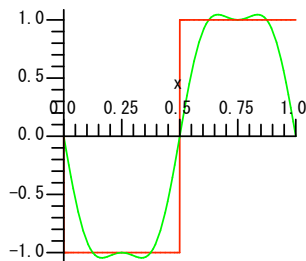
$c_4 := 0.$

$c_5 := -0.828427124 I$

$c_6 := 0.$

$c_7 := -4.828427124 I$

```
> F1:=unapply(sum(evalf(c[i]*exp(I*2*Pi*i/L*x)/N),i=0..(N/2-1))+
sum(evalf(c[N-i]*exp(-I*2*Pi*i/L*x)/N),i=1..(N/2-1)),x):
> plot({Re(F1(x)),F(x)},x=0..1);
```



FFT

```
> x1:=array([evalf(seq(F(i/N),i=0..N-1))]);
  y1:=array([evalf(seq(0,i=0..N-1))]);
          x1:= [ 0. -1. -1. -1. 0. 1. 1. 1. ]
          y1:= [ 0. 0. 0. 0. 0. 0. 0. 0. ]
> FFT(KK,x1,y1);
          8
> print(x1);print(y1);
          [ 0. -1.10^-9 0. 1.10^-9 0. 1.10^-9 0. -1.10^-9 ]
          [ 0. 4.828427122 0. 0.828427124 0. -0.828427124 0. -4.828427122 ]
> F2:=unapply(sum(evalf((x1[i]+I*y1[i])*exp(I*2*Pi*(i-1)/L*x)/N)
,i=1..N/2)+
sum(evalf((x1[N-i+2]+I*y1[N-i+2])*exp(-I*2*Pi*(i-1)/L*x)/N),
i=2..N/2),x):
> plot({Re(F2(x)),F(x)},x=0..1);
```

