-Bairstow-Hitchcock method

Simplest explanation

```
> restart;
```

```
> P:=sort(sum('x^i*a[i]','i'=0..n),x);
```

$$P := \sum_{i=0}^{n} x^i a_i$$

```
> n:=3;
  a:=array(0..n+1,[-40,62,-23,1]):
  sort(P,x);
```

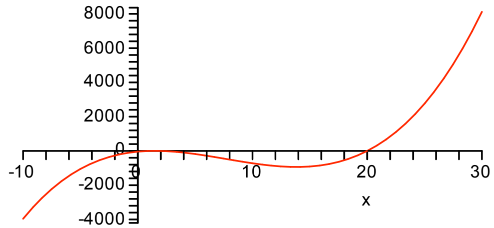
$$n := 3$$

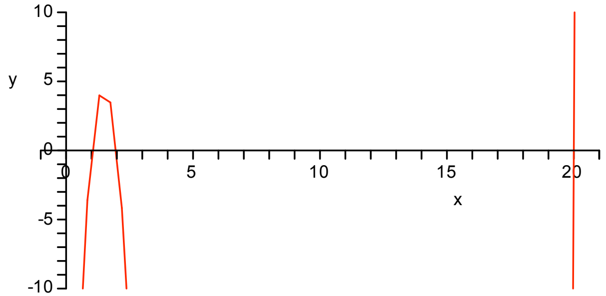
$$x^3 - 23 x^2 + 62 x - 40$$

```
> factor(P);
```

$$(x - 20) (x - 1) (x - 2)$$

```
> plot(P,x=-10..30);
  plot(P,x=-1..21,y=-10..10);
```






```
> fsolve(P=0,x);
```

$$1.000000000, 2.000000000, 20.$$

Analytical solution

Bairstow-Hitchcock法の基本的な考え方の導出.

```
> restart:
  n:=3;
```

$$n := 3$$

```
> P1:=sort(sum(x^i*a[i],i=0..n),x);
```

$$P1 := a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

```
> Q:=sum(x^i*b[i],i=0..n-2);
```

$$Q := b_0 + x b_1$$

```
> P2:=sort(expand((x^2+B*x+C)*Q+R*x+S),x);
```

$$P2 := x^3 + a_2 x^2 + B a_2 x - B^2 x + C x + R x + C a_2 - C B + S$$

二つの式f1とf2を等値して，係数が満たすべき方程式を導出.

```
> for i from 0 to n do
  eq||i:=coeff(P1,x,i)=coeff(P2,x,i);
  end do;
```

$$eq0 := a_0 = C a_2 - C B + S$$

$$eq1 := a_1 = B a_2 - B^2 + C + R$$

$$eq2 := a_2 = a_2$$

$$eq3 := a_3 = 1$$

係数の下の方のbの満たすべき式を導出.

```
> b[n-2]:=1;
  for i from n-3 to 0 by -1 do
  b[i]:=solve(eq||(i+2),b[i]);
  end do;
```

$$b_1 := 1$$

$$b_0 := a_2 - B$$

r,sが０となる方程式をたてる.

```
> Eq_R:=solve(expand(eq1),R)=0;
  Eq_S:=solve(expand(eq0),S)=0;
```

$$Eq\_R := a_1 - B a_2 + B^2 - C = 0$$

$$Eq\_S := a_0 - C a_2 + C B = 0$$

```
> a:=array(0..n,[-40,62,-23,1]);
```

$$a := ARRAY([0 .. 3], [(0) = -40, (1) = 62, (2) = -23, (3) = 1])$$

上のような係数を与えて，p,qを求める.

```
> fsolve({Eq_R,Eq_S},{B,C});
```

$$\{B = -3.000000000, C = 2.000000000\}$$

Numerical solution

数値解を求めるコード

初期値の代入．係数の添え字が逆になっている事に注意．

```
> restart;
  n:=3;
  a:=array(0..n+2,[0,1,-23,62,-40]):
  b:=array(0..n+2,[0,1,0,0,0,0]):
  d:=array(0..n+2,[0,1,0,0,0,0]):
```

$$n := 3$$

```
> B:=1.0;
  C:=1.0;
```

$$B := 1.0$$

$$C := 1.0$$

以下の二つのコマンド領域を収束するまで繰り返す．

```
> for i from 2 to n+2 do
  j:=i-1;
  k:=i-2;
        b[i]:=a[i]-B*b[j]-C*b[k];
        d[i]:=b[i]-B*d[j]-C*d[k];
  od:
```

```
> j:=n-1;
  k:=n-2;
  det1:=d[j]*d[j]-(d[n]-b[n])*d[k];
  dB:=(b[n]*d[j]-b[n+1]*d[k])/det1;
  dC:=(b[n+1]*d[j]-(d[n]-b[n])*b[n])/det1;
  B:=B+dB;
  C:=C+dC;
```

$$j := 2$$

$$k := 1$$

$$det1 := 342.0000000$$

$$dB := -0.$$

$$dC := 0.$$

$$B := -3.000000000$$

$$C := 2.000000000$$

```
> print(convert(b,list));
```

$$[0, 1, -20.00000000, 0., 0., a_5]$$

Note for Numerical solution

det1, dB, とdCはテキスト(3.41)式の逆行列から出てきた式．

```
> with(LinearAlgebra):
```

```
> A:=Matrix(1..2,1..2,[[a11,a12],[a21,a22]]);
```

$$A := \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$$

```
> MatrixInverse(A);
```

$$\begin{bmatrix} \frac{a22}{a11\,a22 - a12\,a21} & -\frac{a12}{a11\,a22 - a12\,a21} \\ -\frac{a21}{a11\,a22 - a12\,a21} & \frac{a11}{a11\,a22 - a12\,a21} \end{bmatrix}$$

ちょっと込み入った例をMapleで扱うと．．．

```
> restart:
  n:=4;
  P:=sort(sum('x^i*a[i]','i'=0..n),x);
  a:=array(0..n+1,[-41,62,-23,1,1]):
  sort(f1,x);
```

$$n := 4$$

$$P := a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$f1$$

```
> fsolve(P=0,x,complex);
```

$$-6.335834399, 1.000000000, 2.167917199 - 1.330888330 I, 2.167917199 + 1.330888330 I$$

```
> x1,x2:=fsolve(P=0,x);
```

$$x1, x2 := -6.335834399, 1.000000000$$

```
> Q:=x^2-(x1+x2)*x+x1*x2;
```

$$Q := x^2 + 5.335834399\,x - 6.335834399$$

```
> R:=factor(P/Q);
```

$$R := x^2 - 4.335834398\,x + 6.471128729$$

```
> expand(R*Q);
```

$$x^4 + 1.000000001\,x^3 - 23.00000000\,x^2 - 41.00000000 + 62.00000000\,x$$