

> restart;

> S1:=-diff(X(x),x,x)+x^2*X(x)-lambda*X(x)=0;

$$S1 := -\left(\frac{d^2}{dx^2} X(x)\right) + x^2 X(x) - \lambda X(x) = 0$$

> X:=x->exp(-x^2/2)*f(x);

$$X := x \rightarrow e^{\left(-\frac{1}{2}x^2\right)} f(x)$$

> S2:=simplify(S1);

$$S2 := -e^{\left(-\frac{1}{2}x^2\right)} \left(-f(x) - 2x \left(\frac{d}{dx} f(x)\right) + \left(\frac{d^2}{dx^2} f(x)\right) + \lambda f(x)\right) = 0$$

> S3:=S2/exp(-1/2*x^2);

$$S3 := f(x) + 2x \left(\frac{d}{dx} f(x)\right) - \left(\frac{d^2}{dx^2} f(x)\right) - \lambda f(x) = 0$$

> D1:=dsolve(S3,f(x),series);

$$D1 := f(x) = f(0) + D(f)(0)x + \left(\frac{1}{2}f(0) - \frac{1}{2}\lambda f(0)\right)x^2 + \left(\frac{1}{2}D(f)(0) - \frac{1}{6}\lambda D(f)(0)\right)x^3 \\ + \left(\frac{5}{24}f(0) - \frac{1}{4}\lambda f(0) + \frac{1}{24}\lambda^2 f(0)\right)x^4 + \left(\frac{7}{40}D(f)(0) - \frac{1}{12}\lambda D(f)(0) + \frac{1}{120}\lambda^2 D(f)(0)\right)x^5 + O(x^6)$$

> factor(coeff(rhs(D1),x,2));

$$-\frac{1}{2}f(0)(-1 + \lambda)$$

> factor(coeff(rhs(D1),x,4));

$$\frac{1}{24}f(0)(-1 + \lambda)(\lambda - 5)$$

> factor(coeff(rhs(D1),x,1));

$$D(f)(0)$$

> factor(coeff(rhs(D1),x,3));

$$-\frac{1}{6}D(f)(0)(-3 + \lambda)$$

> factor(coeff(rhs(D1),x,5));

$$\frac{1}{120}D(f)(0)(-3 + \lambda)(\lambda - 7)$$

>

