

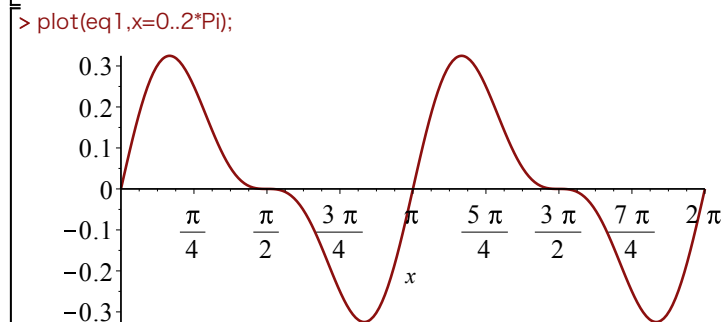
exam17_2_final_ans.mw

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Ex.1

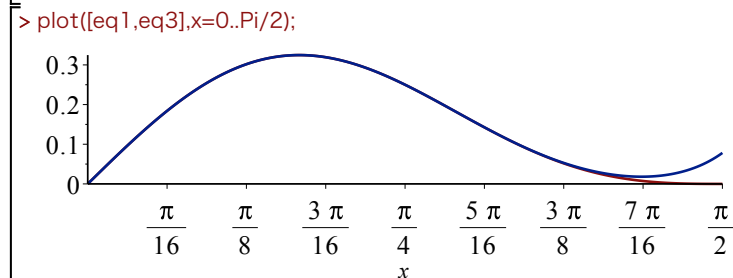
(a) p.44, 7.1(2)

```
> eq1:=sin(x)*cos(x)^3;
eq1 := sin(x) cos(x)^3 (2.1.1)
```



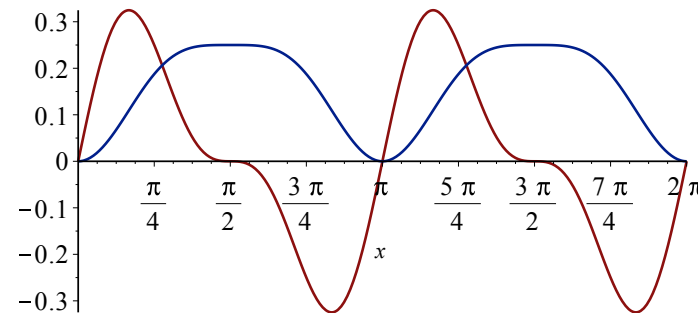
```
> eq2:=series(eq1,x,15);
eq2 := x - 5/3 x^3 + 17/15 x^5 - 26/63 x^7 + 257/2835 x^9 - 82/6237 x^11 + 8194/6081075 x^13 + O(x^15) (2.1.2)
```

```
> eq3:=convert(eq2,polynomial);
eq3 := x - 5/3 x^3 + 17/15 x^5 - 26/63 x^7 + 257/2835 x^9 - 82/6237 x^11 + 8194/6081075 x^13 (2.1.3)
```



```
> eq4:=int(eq1,x=0..x);
eq4 := 1/4 - 1/4 cos(x)^4 (2.1.4)
```

```
> plot([eq1,eq4],x=0..2*Pi);
```



(b) p.89,ex2

```
> assume(x>0);
Int1:=int(sqrt(x^2-1/2*y^2),y=0..x);
Int1 := 1/4 x^2 sqrt(2) + 1/8 x^2 sqrt(2) pi (2.2.1)
```

```
> int(Int1,x=0..1);
1/12 sqrt(2) + 1/24 sqrt(2) pi (2.2.2)
```

Ex.2

(a) p.45, 4-ex3

```
> restart;
with(LinearAlgebra);
v1:=a+b;
v2:=a-b+c;
v3:=a-3*b+2*c;
v1 := a + b
v2 := a - b + c
v3 := a - 3 b + 2 c (3.1.1)
```

```
> eq1:=x*v1+y*v2+z*v3;
eq1 := (a + b) x + (a - b + c) y + (a - 3 b + 2 c) z (3.1.2)
```

```
> eq2:=collect(expand(eq1),{a,b,c});
eq2 := (x + y + z) a + (x - y - 3 z) b + (y + 2 z) c (3.1.3)
```

```
> A:=Matrix([[1,1,1],[1,-1,-3],[0,1,2]]);
A := [ 1 1 1
      1 -1 -3
      0 1 2 ] (3.1.4)
```

```
> A,v:=GenerateMatrix([coeff(eq2,a)=0,coeff(eq2,b)=0,coeff(eq2,c)=0],[x,y,z]);
```

$$A, v := \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -3 \\ 0 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.1.5)$$

```
> Rank(A);
```

$$2 \quad (3.1.6)$$

```
> Determinant(A);
```

$$0 \quad (3.1.7)$$

一次従属

(b) p.60, 4-ex14

```
> x1:=Vector([1,1,0]);
x2:=Vector([1,0,-1]);
x3:=Vector([0,-1,1]);
> y1:=x1;
a1:=y1/sqrt(y1.y1);
```

$$y1 := \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$a1 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \\ 0 \end{bmatrix} \quad (3.2.1)$$

```
> y2:=x2-(x2.a1).a1;
```

$$y2 := \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -1 \end{bmatrix} \quad (3.2.2)$$

```
> a2:=y2/sqrt(y2.y2);
```

(3.2.3)

$$a2 := \begin{bmatrix} \frac{1}{6}\sqrt{6} \\ -\frac{1}{6}\sqrt{6} \\ -\frac{1}{3}\sqrt{6} \end{bmatrix} \quad (3.2.3)$$

```
> y3:=x3-(x3.a1).a1-(x3.a2).a2;
```

$$y3 := \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \quad (3.2.4)$$

```
> a3:=y3/sqrt(y3.y3);
```

$$a3 := \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix} \quad (3.2.5)$$

```
> a1.a2,a1.a1;
a2.a3,a2.a2;
a1.a3,a3.a3;
```

$$\begin{matrix} 0, 1 \\ 0, 1 \\ 0, 1 \end{matrix} \quad (3.2.6)$$

Ex.3 2016_IIB_0_p2

(1)

```
> restart;
c1:=x->1/2*x^2+1/2;
c2:=x->1/4*x^2;
```

$$c1 := x \rightarrow \frac{1}{2}x^2 + \frac{1}{2}$$

$$c2 := x \rightarrow \frac{1}{4}x^2 \quad (4.1.1)$$

```
> S:=int(c1(x)-c2(x),x=a..a+1);
```

$$S := \frac{1}{12}(a+1)^3 - \frac{1}{12}a^3 + \frac{1}{2} \quad (4.1.2)$$

```
> expand(S);
```

$$\frac{1}{4} a^2 + \frac{1}{4} a + \frac{7}{12} \quad (4.1.3)$$

```
> s0:=solve(diff(S,a),a);
```

$$s0 := -\frac{1}{2} \quad (4.1.4)$$

```
> subs(a=s0,S);
```

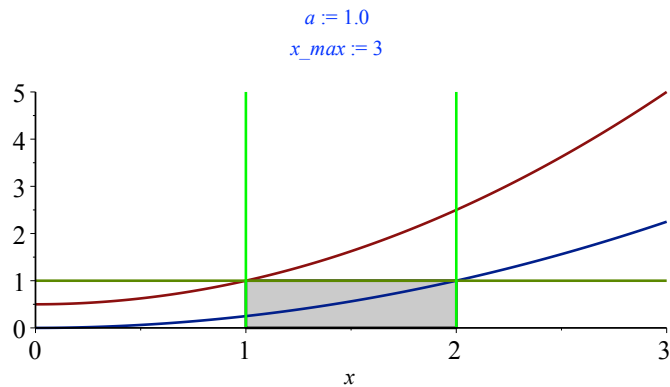
$$\frac{25}{48} \quad (4.1.5)$$

(2)

```
> solve(c1(x)=1,x);
solve(c2(x)=1,x);
```

$$\begin{matrix} 1, -1 \\ 2, -2 \end{matrix} \quad (4.2.1)$$

```
> with(plots):with(plottools);
> a:=1.0;x_max:=3;
p1:=plot([c1(x),c2(x),1],x=0..x_max);
l1:=line([a,0],[a,c1(x_max)],color=green);
l2:=line([a+1,0],[a+1,c1(x_max)],color=green);
rect:=rectangle([a,0],[a+1,1],color=gray);
display(p1,l1,l2,rect);
```



```
> a:='a';
Su:=int(c1(x)-1,x=1..a+1);
```

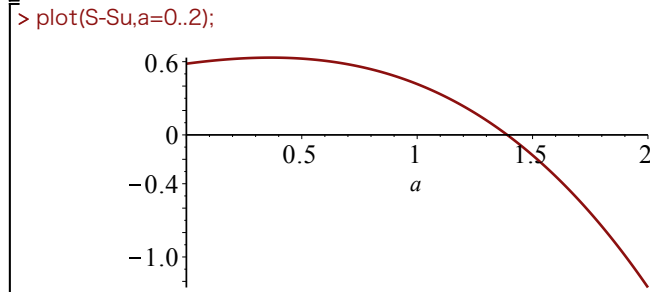
$$a := a$$

$$Su := \frac{1}{6} (a + 1)^3 - \frac{1}{6} - \frac{1}{2} a \quad (4.2.2)$$

```
> expand(Su);
```

$$\frac{1}{6} a^3 + \frac{1}{2} a^2 \quad (4.2.3)$$

```
> expand(S-Su);
```

$$-\frac{1}{6} a^3 - \frac{1}{4} a^2 + \frac{1}{4} a + \frac{7}{12} \quad (4.2.4)$$


```
> solve(diff(S-Su,a),a);
```

$$-\frac{1}{2} - \frac{1}{2} \sqrt{3}, \frac{1}{2} \sqrt{3} - \frac{1}{2} \quad (4.2.5)$$

Ex.4 2016_IIB_0_p2_rev

(1)

```
> restart;
c1:=x->0.5*x^2+0.5;
c2:=x->0.1*x^2;
```

$$\begin{matrix} c1 := x \rightarrow 0.5 x^2 + 0.5 \\ c2 := x \rightarrow 0.1 x^2 \end{matrix} \quad (5.1.1)$$

```
> S:=int(c1(x)-c2(x),x=a..a+1);
```

$$S := 0.1333333333 (a + 1)^3 - 0.1333333333 a^3 + 0.5000000000 \quad (5.1.2)$$

```
> expand(S);
```

$$0.6333333333 + 0.3999999999 a^2 + 0.3999999999 a \quad (5.1.3)$$

```
> s0:=solve(diff(S,a),a);
```

$$s0 := -0.5000000000 \quad (5.1.4)$$

```
> subs(a=s0,S);
```

$$0.5333333333 \quad (5.1.5)$$

(2)

```
> solve(c1(x)=1,x);
s2:=solve(c2(x)=1,x);
```

$$1., -1.$$

```
s2 := 3.162277660, -3.162277660
```

(5.2.1)

```
> x_2:=s2[1];
```

```
x_2 := 3.162277660
```

(5.2.2)

```
> with(plots):with(plottools):
```

```
> a:=1.05;x_max:=4;
```

```
p1:=plot([c1(x),c2(x),1],x=0..x_max):
```

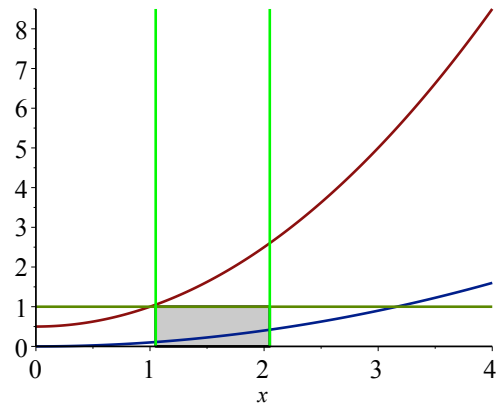
```
l1:=line([a,0],[a,c1(x_max)],color=green):
```

```
l2:=line([a+1,0],[a+1,c1(x_max)],color=green):
```

```
rect:=rectangle([a,0],[a+1,1],color=gray):
```

```
display(p1,l1,l2,rect);
```

```
a := 1.05
x_max := 4
```



```
> a:='a';
```

```
Su:=int(c1(x)-1,x=1..a+1);
```

```
a := a
```

```
Su := 0.1666666667 (a + 1.)3 - 0.1666666667 - 0.5000000000 a
```

(5.2.3)

```
> expand(Su);
```

```
0.1666666667 a3 + 0.5000000001 a2 + 1. 10-10 a
```

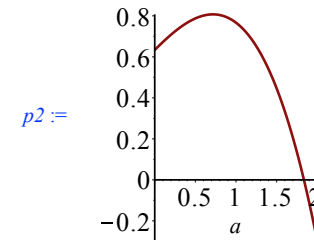
(5.2.4)

```
> expand(S-Su);
```

```
-0.1666666667 a3 - 0.1000000002 a2 + 0.3999999998 a + 0.6333333333
```

(5.2.5)

```
> p2:=plot(S-Su,a=0..2);
```



```
> solve(diff(S-Su,a),a);
```

```
-1.116515139, 0.7165151384
```

(5.2.6)

補足

実際のTの変動は以下の通り.

```
> S3:=int(1-c2(x),x=a..a+1);
```

```
S4:=int(1-c2(x),x=a..x_2);
```

```
S3 := 1. - 0.03333333333 (a + 1.)3 + 0.03333333333 a3
```

```
S4 := 2.108185107 - 1. a + 0.03333333333 a3
```

(5.3.1)

```
> p3:=plot(S3,a=1..x_2-1,color=blue);
```

```
p4:=plot(S4,a=x_2-1..x_2,color=green);
```

```
display(p2,p3,p4);
```

