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線形代数

基底，次元，成分

$\boldsymbol{R}^3$ において $a_1 = (2, -1, 0), a_2 = (1, 0, 1), a_3 = (1, 2, -2)$ は基底をなす． $a = (-4, -2, 1)$ の基底 $\mathcal{B} = \{a_1, a_2, a_3\}$ に関する成分を求めよ．

In [32]:
```
from sympy import *
init_session()

A=Matrix([[2,1,1,-4],[-1,0,2,-2],[0,1,-2,1]])
A
```

```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

Out[32]:
$$\begin{bmatrix} 2 & 1 & 1 & -4 \\ -1 & 0 & 2 & -2 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

In [33]:
```
A.rref()
```

Out[33]:
$$\left(\begin{bmatrix} 1 & 0 & 0 & -\frac{4}{7} \\ 0 & 1 & 0 & -\frac{11}{7} \\ 0 & 0 & 1 & -\frac{9}{7} \end{bmatrix}, \quad [0, \quad 1, \quad 2]\right)$$

Ker, Im

$A = \begin{pmatrix} 1 & 0 & -1 & -2 \\ -2 & 1 & 3 & 5 \\ 1 & 1 & 0 & -1 \end{pmatrix}$ とする． R^4 から R^3 への線形写像 f を $f(x) = Ax$ で 与えるとき，f の Imf および Ker$f$ の次元と1組の基底を求めよ．

In [34]:
```
from sympy import *
init_session()
A=Matrix([[1,0,-1,-2],[-2,1,3,5],[1,1,0,-1]])
A
```

```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

Out[34]:
$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ -2 & 1 & 3 & 5 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

In [35]:
```
A.rref()
```

Out[35]:
$$\left(\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad [0, \quad 1]\right)$$

In [36]:
```
A.nullspace()
```

Out[36]: $\left[\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right]$

微積分

正規分布の概形

関数

$$f(x) = e^{-x^2}$$

の増減，極値，凹凸を調べ，曲線$y = f(x)$の概形を描け．

In [1]:
```
from sympy import *
init_session()

f = exp(-x**2)
f
```

```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

Out[1]: e^{-x^2}

In [2]:
```
df = f.diff(x)
df
```

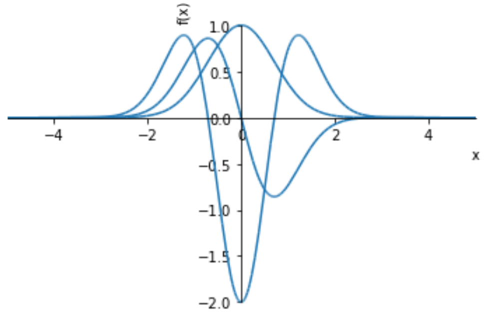
Out[2]: $-2xe^{-x^2}$

In [3]:
```
df2 = f.diff(x,x)
df2
```

Out[3]: $2\left(2x^2 - 1\right) e^{-x^2}$

In [4]:
```
%matplotlib inline

plot(f,df,df2,(x,-5,5))
```



Out[4]: <sympy.plotting.plot.Plot at 0x11a8d6828>

In [5]:
```
solve(df,x)
```

Out[5]: $[0]$

In [6]:
```
solve(df2,x)
```

Out[6]: $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$

x	$-\infty$	···	$-\frac{\sqrt{2}}{2}$	···	**0**	···	$\frac{\sqrt{2}}{2}$	···	∞
f(x)	0	↗	↗	↗	1.0	↘	↘	↘	0
f'(x)	0	+	+	+	0	-	-	-	0
f''(x)	0	+	0	-	-	-	0	+	0

積分

関数

$$f(x) = \frac{1}{\cos x + 1}$$

の不定積分を求めよ．また，$x = 0..\pi/2$の定積分を求めよ．

In [18]:
```
from sympy import *
init_session()

1/(cos(x)+1)
```

```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

Out[18]: $\frac{1}{\cos(x)+1}$

In [15]:
```
integrate(1/(cos(x)+1),x)
```

Out[15]: $\tan\left(\frac{x}{2}\right)$

In [6]:
```
integrate(1/(cos(x)+1),(x,0,pi/2))
```

Out[6]: 1

センター試験原題

(2017大学入試センター試験　追試験 数学II・B 第2問)

関数$f(x)=x^3-5x^2+3x-4$について考える．関数$f(x)$の増減を調べよう．$f(x)$の導関数は

$$f'(x)=\boxed{ア}x^2-\boxed{イウ}x+\boxed{エ}$$

であり，$f(x)$は$x=\frac{\boxed{オ}}{\boxed{カ}}$で極大値，$x=\boxed{キ}$で極小値をとる．よって，$x\geqq 0$の範囲における$f(x)$の最小値は$\boxed{クケコ}$である．

また，方程式$f(x)=0$の異なる実数解の個数は$\boxed{サ}$個である．

In [4]:
```
from sympy import *
init_session()
```


```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```


In [67]:
```
aa=1
f= aa*x**3-5*x**2+3*x-4
f
```

Out[67]: x^3-5x^2+3x-4

In [68]:
```
df = diff(f,x)
df
```

Out[68]: $3x^2-10x+3$

In [69]:
```
s1=solve(df,x, dict=true)
s1
```

Out[69]: $\left[\left\{x:\frac{1}{3}\right\},\quad\{x:3\}\right]$

In [70]:
```
f.subs(s1[0])
```

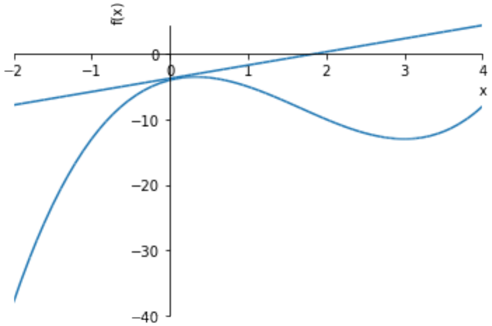
Out[70]: $-\frac{95}{27}$

In [71]:
```
f.subs(s1[1])
```

Out[71]: -13

In [72]:
```
%matplotlib inline

from sympy import *
plot(f,yl,(x,-2,4))
```



Out[72]: <sympy.plotting.plot.Plot at 0x1170402e8>

In [77]:
```
x0=0
m = df.subs({x:x0})
yl=m*x+f.subs({x:x0})
yl
```

Out[77]: $3x-4$

2

曲線$y=f(x)$上の点$(0,f(0))$における接線をlとすると，lの方程式は$y=$ [シ] $x-$ [ス] である．また，放物線$y=x^2+px+q$をCとし，Cは点(a, [シ] $a-$ [ス])で lと接しているとする．このとき，p,qはaを用いて

$p=$ [セソ] $a+$ [タ] , $q=a^{[チ]}-$ [ツ]

と表される．

In [78]:
```
a, p, q = symbols('a, p,q')
g=x**2+p*x+q
g
```

Out[78]: $px+q+x^2$

In [79]:
```
eq1=g.subs({x:a})-yl.subs({x:a})
eq1
```

Out[79]: $a^2+ap-3a+q+4$

In [80]:
```
eq2=g.diff(x).subs({x:a})-m
eq2
```

Out[80]: $2a+p-3$

In [81]:
```
s2=solve(eq2,p,dict=true)
s2[0]
```

Out[81]: $\{p:-2a+3\}$

In [82]:
```
solve(eq1.subs(s2[0]),q)
```

Out[82]: $[a^2-4]$

数値改変

問3.において，関数$f(x)=1.1x^3-5x^2+3x-4$，また，曲線$y=f(x)$上の点$(0.1,f(0.1))$における接線をlとして問題を解け．[ツ] は3.7489となる．

In [1]:
```
from sympy import *
init_session()
```

```
IPython console for SymPy 1.0 (Python 3.6.1-64-bit) (ground types:
python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

In [2]:
```
aa=1.1
f= aa*x**3-5*x**2+3*x-4
f
```

Out[2]: $1.1x^3-5x^2+3x-4$

In [3]:
```
df = diff(f,x)
df
```

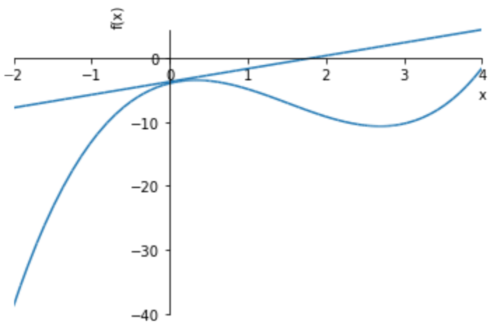
Out[3]: $3.3x^2-10x+3$

In [4]:
```
s1=solve(df,x, dict=true)
s1
```
Out[4]: $[\{x : 0.337614592259064\}, \quad \{x : 2.69268843804397\}]$

In [5]:
```
f.subs(s1[0])
```
Out[5]: -3.51474350365896

In [6]:
```
f.subs(s1[1])
```
Out[6]: -10.6989081645382

In [28]:
```
%matplotlib inline

from sympy import *
plot(f,yl,(x,-2,4))
```


Out[28]: <sympy.plotting.plot.Plot at 0x11ee4d0b8>

In [29]:
```
x0=0.1
m = df.subs({x:x0})
#yl=m*x+f.subs({x:x0}) # HERE!!!
yl=m*(x-x0)+f.subs({x:x0})
yl
```
Out[29]: $2.033x - 3.9522$

In [30]:
```
a, p,q = symbols('a, p,q')
g=x**2+p*x+q
g
```
Out[30]: $px + q + x^2$

In [31]:
```
eq1=g.subs({x:a})-yl.subs({x:a})
eq1
```
Out[31]: $a^2 + ap - 2.033a + q + 3.9522$

In [32]:
```
eq2=g.diff(x).subs({x:a})-m
eq2
```
Out[32]: $2a + p - 2.033$

In [33]:
```
s2=solve(eq2,p,dict=true)
s2[0]
```
Out[33]: $\{p : -2.0a + 2.033\}$

In [34]:
```
solve(eq1.subs(s2[0]),q)
```
Out[34]: $\left[a^2 - 3.9522\right]$

In [ ]:
```
```