1(数値解の収束性)

```
> restart;
  func:=x->-3*exp(-x)+exp(-3*x);
```

$$func := x \rightarrow -3\,e^{-x} + e^{-3\,x} \quad \textbf{(1.1)}$$

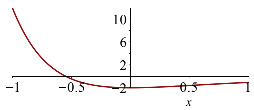
```
> s1:=solve(func(x),x);
```

$$s1 := -\frac{1}{2}\ln(3) \quad \textbf{(1.2)}$$

```
> x0:=evalf(s1);
```

$$x0 := -0.5493061445 \quad \textbf{(1.3)}$$

```
> plot(func(x),x=-1..1);
```



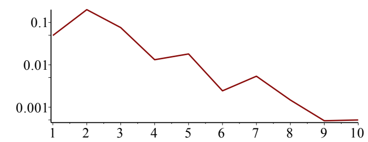

```
> x1:=-1.0: x2:=0.0: res1:=[];
>   f1:=func(x1): f2:=func(x2):
>   for i from 1 to 10 do
>     x:=(x1+x2)/2;
>     f:=func(x);
>     if f*f1>=0.0 then
>       x1:=x; f1:=f;
>     else
>       x2:=x; f2:=f;
>     end if;
>     printf("%20.15f, %20.15f\n",x,f);
  res1:=[op(res1),[i,abs(x-x0)]];
>   end do:
```

$$res1 := [\,]$$

```
-0.500000000000000,   -0.464474743000000
-0.750000000000000,    3.136735785000000
-0.625000000000000,    0.916081249000000
-0.562500000000000,    0.140784954000000
-0.531250000000000,   -0.180999397000000
-0.546875000000000,   -0.025142671000000
-0.554687500000000,    0.056530114000000
-0.550781250000000,    0.015375046000000
-0.548828125000000,   -0.004962977000000
-0.549804687500000,    0.005186178000000
```

```
> res1;
  with(plots):
  logplot(res1);
```

[[1, 0.0493061445], [2, 0.2006938555], [3, 0.0756938555], [4, 0.0131938555], [5, 0.0180561445], [6, 0.0024311445], [7, 0.0053813555], [8, 0.0014751055], [9, 0.0004780195], [10, 0.0004985430]]



2 (Gauss-Seidelの収束性)

```
> restart;
  Digits:=10;
  with(LinearAlgebra):
```

$$Digits := 10 \quad \textbf{(2.1)}$$

```
> GaussSeidel:=proc(t,irep)
  local AA,b,n,x0,x1,iter,i,j;
  AA:=Matrix([[1,t,t],[t,1,t],[t,t,1]]):
>   b:=Vector([2,2,2]): n:=3;
  print(evalf(MatrixInverse(AA).b));
>   x0:=[0,0,0]: x1:=[0,0,0]:
>   for iter from 1 to irep do
>     for i from 1 to n do
>       x1[i]:=b[i];
>       for j from 1 to n do
>         x1[i]:=x1[i]-AA[i,j]*x0[j];
>       end do:
>       x1[i]:=x1[i]+AA[i,i]*x0[i];
>       x0[i]:=evalf(x1[i]/AA[i,i]);
>     end do:
> #     x0:=evalf(x1);
>     print(iter,x0);
>   end do:
> end proc:
```

```
> GaussSeidel(0.2,10); #6あたり
  GaussSeidel(0.5,17); #15あたり
  GaussSeidel(0.9,100); #96あたり
```

$$\begin{bmatrix} 1.42857142857143 \\ 1.42857142857143 \\ 1.42857142857143 \end{bmatrix}$$

1, [2., 1.6, 1.28]

2, [1.424, 1.4592, 1.42336]

3, [1.423488, 1.4306304, 1.42917632]

4, [1.428038656, 1.428557005, 1.428680868]

5, [1.428552425, 1.428553341, 1.428578847]

6, [1.428573562, 1.428569519, 1.428571384]

7, [1.428571819, 1.428571359, 1.428571364]

8, [1.428571455, 1.428571436, 1.428571422]
9, [1.428571428, 1.428571430, 1.428571428]
10, [1.428571428, 1.428571428, 1.428571428]

$$\begin{bmatrix} 1. \\ 1. \\ 1. \end{bmatrix}$$

1, [2., 1.0, 0.50]
2, [1.250, 1.1250, 0.81250]
3, [1.031250, 1.0781250, 0.94531250]
4, [0.988281250, 1.033203125, 0.9892578125]
5, [0.9887695313, 1.010986328, 1.000122070]
6, [0.9944458013, 1.002716064, 1.001419067]
7, [0.9979324348, 1.000324250, 1.000871658]
8, [0.9994020458, 0.9998631480, 1.000367403]
9, [0.9998847243, 0.9998739365, 1.000120670]
10, [1.000002697, 0.9999383170, 1.000029494]
11, [1.000016094, 0.9999772060, 1.000003350]
12, [1.000009722, 0.9999934640, 0.9999984070]
13, [1.000004064, 0.9999987645, 0.9999985858]
14, [1.000001325, 1.000000045, 0.9999993155]
15, [1.000000320, 1.000000182, 0.9999997490]
16, [1.000000034, 1.000000108, 0.9999999290]
17, [0.9999999815, 1.000000044, 0.9999999870]

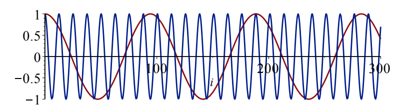
$$\begin{bmatrix} 0.714285714285714 \\ 0.714285714285714 \\ 0.714285714285714 \end{bmatrix}$$

1, [2., 0.2, 0.02]
2, [1.802, 0.3602, 0.05402]
3, [1.627202, 0.4869002, 0.09730802]
4, [1.474212602, 0.585631440, 0.1461403620]
5, [1.341405378, 0.6612088342, 0.1976472092]
6, [1.227029561, 0.7177909067, 0.2496615790]
7, [1.129292763, 0.7589410919, 0.3005895303]
8, [1.046422440, 0.7876892267, 0.3492995000]
90, [0.7142856624, 0.7142865744, 0.7142849870]
91, [0.7142855951, 0.7142864757, 0.7142851359]
92, [0.7142855497, 0.7142863827, 0.7142852606]
93, [0.7142855208, 0.7142862965, 0.7142853642]
94, [0.7142855052, 0.7142862172, 0.7142854495]
95, [0.7142855001, 0.7142861454, 0.7142855191]
96, [0.7142855020, 0.7142860808, 0.7142855753]
97, [0.7142855095, 0.7142860232, 0.7142856201]
98, [0.7142855205, 0.7142859739, 0.7142856555]
99, [0.7142855340, 0.7142859290, 0.7142856829]
100, [0.7142855493, 0.7142858914, 0.7142857037] **(2.2)**

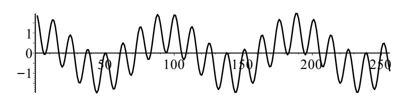
3(FFT の強度表示)

```
> restart:
  funcs:=[cos(i/15),cos(i/2)];
  plot(funcs,i=0..300);
```

$$funcs := \left[\cos\left(\frac{1}{15}\,i\right), \cos\left(\frac{1}{2}\,i\right)\right]$$

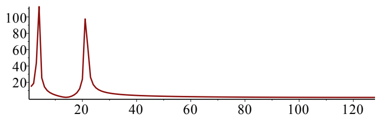



```
> data1:=[]:
>   for i from 1 to 256 do
>     data1:=[op(data1),evalf(funcs[1]+funcs[2])];
>   end do:
>   with(plots):
>   listplot(data1);
```



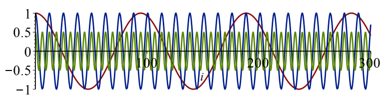

```
> X:=array(data1):
> Y:=array(1..256,sparse):
> FFT(8,X,Y);
> Data2:=[seq([i,sqrt(X[i]^2+Y[i]^2)],i=1..128)]:
> plot(Data2);
```

256

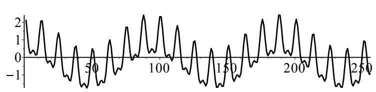



```
> restart:
  funcs:=[cos(i/15),cos(i/2),1/2*cos(i)];
  plot(funcs,i=0..300);
```

$$funcs := \left[\cos\left(\frac{1}{15}\,i\right), \cos\left(\frac{1}{2}\,i\right), \frac{1}{2}\,\cos(i)\right]$$

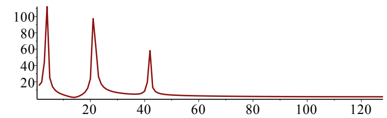



```
> data1:=[]:
>   for i from 1 to 256 do
>     data1:=[op(data1),evalf(funcs[1]+funcs[2]+funcs[3])];
>   end do:
>   with(plots):
>   listplot(data1);
```




```
> X:=array(data1):
> Y:=array(1..256,sparse):
> FFT(8,X,Y);
> Data2:=[seq([i,sqrt(X[i]^2+Y[i]^2)],i=1..128)]:
> plot(Data2);
```

$$256$$



4(正接(tan)関数のニュートンの差分商補間)

```
> restart;
  Digits:=7;
  func:=x->tan(x);
```

$$Digits := 7$$

$$func := x \rightarrow \tan(x) \qquad \textbf{(4.1)}$$

```
> evalf(tan(Pi/4));
```

$$1. \qquad \textbf{(4.2)}$$

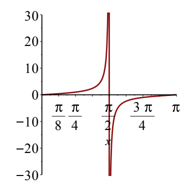
```
> tan(Pi/4);
```

$$1 \qquad \textbf{(4.3)}$$

```
> evalf(Pi/4);
```

$$0.7853982 \qquad \textbf{(4.4)}$$

```
> plot(func(x),x=0..Pi);
```




```
> x[1]:=0.6;x[2]:=0.7;x[3]:=0.8;x[4]:=0.9;
```

$$x_1 := 0.6$$

$$x_2 := 0.7$$

$$x_3 := 0.8$$

$$x_4 := 0.9 \qquad \textbf{(4.5)}$$

```
> f[0][1]:=func(x[1]);
  f[0][2]:=func(x[2]);
  f[0][3]:=func(x[3]);
  f[0][4]:=func(x[4]);
```

$f_{0_1} := 0.6841368$

$f_{0_2} := 0.8422884$

$f_{0_3} := 1.029639$

$f_{0_4} := 1.260158$ **(4.6)**

```
> f[1][1]:=(f[0][2]-f[0][1])/(x[2]-x[1]);
  f[1][2]:=(f[0][3]-f[0][2])/(x[3]-x[2]);
  f[1][3]:=(f[0][4]-f[0][3])/(x[4]-x[3]);
```

$f_{1_1} := 1.581516$

$f_{1_2} := 1.873506$

$f_{1_3} := 2.305190$ **(4.7)**

```
> f[2][1]:=(f[1][2]-f[1][1])/(x[3]-x[1]);
  f[2][2]:=(f[1][3]-f[1][2])/(x[4]-x[2]);
```

$f_{2_1} := 1.459950$

$f_{2_2} := 2.158420$ **(4.8)**

```
> f[3][1]:=(f[2][2]-f[2][1])/(x[4]-x[1]);
```

$f_{3_1} := 2.328233$ **(4.9)**

```
> n:=4;
  for m from 1 to n-1 do
  for j from 1 to n-m do
    f[m][j]:=(f[m-1][j+1]-f[m-1][j])/(x[j+m]-x[j]);
    print(m,j,f[m][j]);
  end;
  end;
```

$n := 4$

1, 1, 1.581516

1, 2, 1.873506

1, 3, 2.305190

2, 1, 1.459950

2, 2, 2.158420

3, 1, 2.328233 **(4.10)**

1

```
> f[2][2];
```

2.158420 **(4.1.1)**

2

```
> xx:=0.7853982;
  p[1]:=f[0][1]+(xx-x[1])*f[1][1];
```

$xx := 0.7853982$

$p_1 := 0.9773470$ **(4.2.1)**

```
> p[2]:=p[1]+(xx-x[1])*(xx-x[2])*f[2][1];
```

$p_2 := 1.000462$ **(4.2.2)**

3

```
> p[3]:=p[2]+(xx-x[1])*(xx-x[2])*(xx-x[3])*f[3][1];
```

$p_3 := 0.9999237$ **(4.3.1)**

```
> evalf(func(xx));
```

1.000000 **(4.11)**

```
> evalf(tan(Pi/4));
```

1. **(4.12)**