2014年度　数値計算(西谷) 解答例

1

```
> restart;
  int(4/(1+x^2),x=0..1);
```

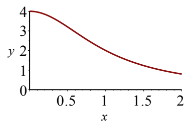
$$\pi \qquad (2.1)$$

```
> func:=x->4/(1+x^2);
  yy:=[0,0,0];
```

$$func := x \rightarrow \frac{4}{x^2+1}$$

$$yy := [0, 0, 0] \qquad (2.2)$$

```
> plot(func(x),x=0..2,y=0..4);
```




```
> y1:=(func(0)+func(1))/2*1.0;
  yy[1]:=[1,y1-evalf(Pi)];
```

$$y1 := 3.000000000$$

$$yy_1 := [1, -0.141592654] \qquad (2.3)$$

```
> y2:=(func(0)/2+func(0.5)+func(1)/2)*0.5;
  yy[2]:=[2,y2-evalf(Pi)];
```

$$y2 := 3.100000000$$

$$yy_2 := [2, -0.041592654] \qquad (2.4)$$

```
> y3:=(func(0)/2+func(0.25)+func(0.5)+func(0.75)+func(1)
  /2)*0.25;
  yy[3]:=[4,y3-evalf(Pi)];
```

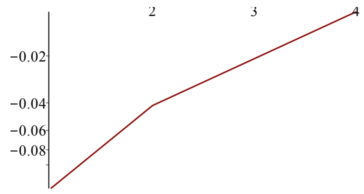
$$y3 := 3.131176470$$

$$yy_3 := [4, -0.010416184] \qquad (2.5)$$

```
> print(yy);
```

$$[[1, -0.141592654], [2, -0.041592654], [4, -0.010416184]] \qquad (2.6)$$

```
> with(plots):
  logplot(yy);
```



精確には，absによって，あるいは引き算の順序を換え，誤差は絶対値を取るべき．

2

```
> restart;
  a:=0.105360;
  b:=0.117783;
  c:=2.0;
  den:=a-b;
```

$$a := 0.105360$$

$$b := 0.117783$$

$$c := 2.0$$

$$den := -0.012423 \qquad (3.1)$$

```
> Digits:=5;
  den:=a-b;
  den/c;
```

$$Digits := 5$$

$$den := -0.01242$$

$$-0.0062100 \qquad (3.2)$$

```
> Digits:=3;
  den:=a-b;
  den/c;
```

$$Digits := 3$$

$$den := -0.013$$

$$-0.00650 \qquad (3.3)$$

```
> Digits:=2;
  den:=a-b;
  den/c;
```

$$Digits := 2$$

$$den := -0.01$$

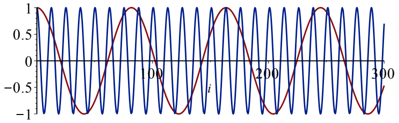
$$-0.0050 \qquad (3.4)$$

[3]

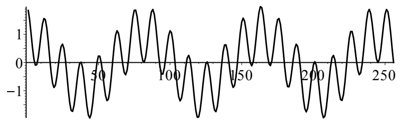
1

```
> restart:
  funcs:=[cos(i/13),cos(i/2)];
  #funcs:=[cos(i/13),1/5*cos(2*i)];
  plot(funcs,i=0..300);
```

$$funcs := \left[\cos\left(\frac{1}{13}\,i\right), \cos\left(\frac{1}{2}\,i\right)\right]$$



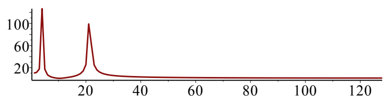

```
> data1:=[]:
>   for i from 1 to 256 do
>     data1:=[op(data1),evalf(funcs[1]+funcs[2])];
>   end do:
>   with(plots):
>   listplot(data1);
```




```
> X:=array(data1):
> Y:=array(1..256,sparse):
> FFT(8,X,Y);
```

$$256 \tag{4.1.1}$$

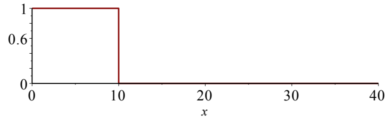
```
> Data2:=[seq([i,sqrt(X[i]^2+Y[i]^2)],i=1..128)]:
> plot(Data2);
```



2

```
> filter:=x->piecewise(x>=0 and x<=10,1); #方形フィルタ
> plot(filter(x),x=0..40);
```

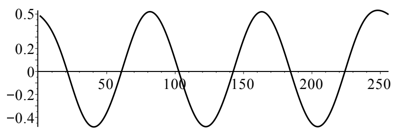
$$filter := x \rightarrow piecewise(0 \le x \textbf{ and } x \le 10, 1)$$




```
> FRdata:=array([seq(X[i]*filter(i),i=1..256)]):
> FIdata:=array([seq(Y[i]*filter(i),i=1..256)]):
> iFFT(8,FRdata,FIdata);
```

$$256 \tag{4.2.1}$$

```
> listplot(FRdata);
```



[4]対数関数のニュートンの差分商近似

```
> restart;
  Digits:=7;
  x[1]:=8.0;x[2]:=9.0;x[3]:=10.0;x[4]:=11.0;
```

$$Digits := 7$$
$$x_1 := 8.0$$
$$x_2 := 9.0$$
$$x_3 := 10.0$$
$$x_4 := 11.0 \tag{5.1}$$

```
> f[0][1]:=ln(x[1]);
  f[0][2]:=ln(x[2]);
  f[0][3]:=ln(x[3]);
  f[0][4]:=ln(x[4]);
```

$$f_{0_1} := 2.079442$$

$$f_{0_2} := 2.197225$$

$$f_{0_3} := 2.302585$$

$$f_{0_4} := 2.397895 \quad \textbf{(5.2)}$$

```
> f[1][1]:=(f[0][2]-f[0][1])/(x[2]-x[1]);
  f[1][2]:=(f[0][3]-f[0][2])/(x[3]-x[2]);
  f[1][3]:=(f[0][4]-f[0][3])/(x[4]-x[3]);
```

$$f_{1_1} := 0.1177830$$

$$f_{1_2} := 0.1053600$$

$$f_{1_3} := 0.09531000 \quad \textbf{(5.3)}$$

```
> f[2][1]:=(f[1][2]-f[1][1])/(x[3]-x[1]);
  f[2][2]:=(f[1][3]-f[1][2])/(x[4]-x[2]);
```

$$f_{2_1} := -0.006211500$$

$$f_{2_2} := -0.005025000 \quad \textbf{(5.4)}$$

```
> f[3][1]:=(f[2][2]-f[2][1])/(x[4]-x[1]);
```

$$f_{3_1} := 0.0003955000 \quad \textbf{(5.5)}$$

```
> n:=4;
  for m from 1 to n-1 do
  for j from 1 to n-m do
    f[m][j]:=(f[m-1][j+1]-f[m-1][j])/(x[j+m]-x[j]);
    print(m,j,f[m][j]);
  end;
  end;
```

$n := 4$

1, 1, 0.1177830

1, 2, 0.1053600

1, 3, 0.09531000

2, 1, -0.006211500

2, 2, -0.005025000

3, 1, 0.0003955000 **(5.6)**

1

```
> 2, 1, -0.6211500e-2;
```

2, 1, -0.006211500 **(5.1.1)**

2

```
> xx:=9.2;
  p[1]:=f[0][1]+(xx-x[1])*f[1][1];
```

$xx := 9.2$

$$p_1 := 2.220782 \quad \textbf{(5.2.1)}$$

```
> p[2]:=p[1]+(xx-x[1])*(xx-x[2])*f[2][1];
```

$$p_2 := 2.219291 \quad \textbf{(5.2.2)}$$

3

```
> p[3]:=p[2]+(xx-x[1])*(xx-x[2])*(xx-x[3])*f[3][1];
```

$$p_3 := 2.219215 \quad \textbf{(5.3.1)}$$

```
> evalf(ln(xx));
```

2.219203 **(5.7)**