# 線形代数 演習-IV

22/10/20

## ○写像

$v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

変換行列

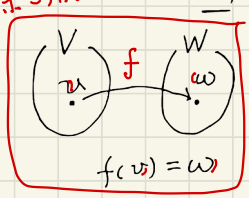
$A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$

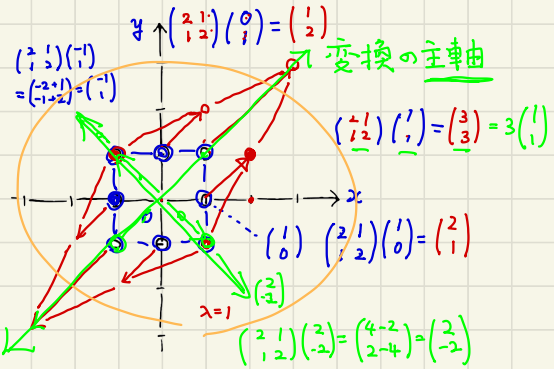
$w_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $w_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$

ホニョ絵




$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2+1 \\ -1+2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4-2 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

## ○固有値 (Eigen Value)

$\begin{pmatrix} x \\ y \end{pmatrix}$

$\Rightarrow A v = \lambda v$

$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$|A - \lambda E|$

$= \left| \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right|$

$= \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$= (2-\lambda)^2 - 1$

$= 4 - 4\lambda + \lambda^2 - 1$

$= \lambda^2 - 4\lambda + 3$

$\lambda = 1, 3$

$= (\lambda - 3)(\lambda - 1)$

行列式の基本操作

$\Rightarrow$① $\lambda_1 = 1$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1 \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

$2x + y = x$

$x + 2y = y$

$\Rightarrow$② $x + y = 0$

$x + y = 0$

$v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\Rightarrow$③

$1\ 1\ 0$

$-\ 1\ 1\ 0$

$1\ 1\ 0$

$0\ 0\ 0$

検算?

$\Rightarrow$④ $\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$x + \lambda = 0$

$x = -\lambda$

階段になるので.

$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

## ○固有ベクトル (Eigen Vectors)

$\lambda_2 = 3$

$v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = 3\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$Av = \lambda v$

$\begin{pmatrix} -2+1 \\ -1+2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$