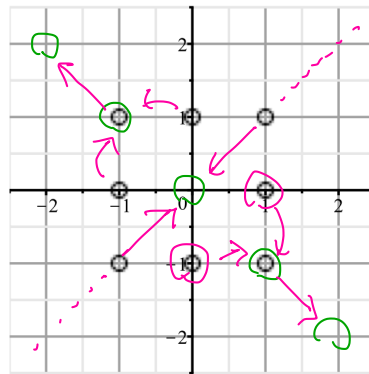


情報工学のための数学演習 (線形代数) 試験問題

1. 表現行列を $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ とする写像によって, 図1の丸で示した8点はどこへ写像されるか? 解答用紙に図1を書き写して写像前後の点をプロットせよ. また, この写像の核 Ker(Kernel) の基底を求めよ. (20点)



$$\text{Im } f: y = -x$$

$$\text{Ker } f: y = x$$

$$\begin{pmatrix} 1 & 0 & -7/4 \\ 0 & 1 & -5/4 \end{pmatrix}$$

$$7x_1 + 5x_2 + 4x_3 = 0$$

図1: 写像プロット.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -5/7 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -4/7 \\ 0 \\ 1 \end{pmatrix}$$

2. $V = \{(x_1, x_2, x_3) \in \mathbf{R}^3; 2x_1 - 2x_2 - x_3 = 0, x_1 - 3x_2 + 2x_3 = 0\}$ の直交補空間 V^\perp を求めよ. (20点)
3. \mathbf{R}^3 のベクトル $\mathbf{a} = (-2, 1, 2)$, $\mathbf{b} = (2, -4, 2)$ を, ベクトル $\mathbf{a}_1 = (1, -1, 1)$, $\mathbf{a}_2 = (-1, -1, -1)$, $\mathbf{a}_3 = (-1, -3, -1)$ の一次結合で表せ. (20点)
4. 次の行列の固有値とそれに対する固有空間を求めよ. (20点)

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix}$$

$$\lambda = 4 \quad 2 \quad -2$$

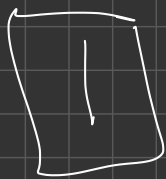
$$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

5. $A = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 2 & -1 & -1 & 3 \\ -1 & 1 & 0 & -2 \end{pmatrix}$ とする. \mathbf{R}^4 から \mathbf{R}^3 への線形写像 f を $f(\mathbf{x}) = A\mathbf{x}$ で与えるとき, f の $\text{Im } f$ および $\text{Ker } f$ の次元と1組の基底を求めよ. (20点)

$$\text{Im } f \quad 2 \text{ dim} \quad \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

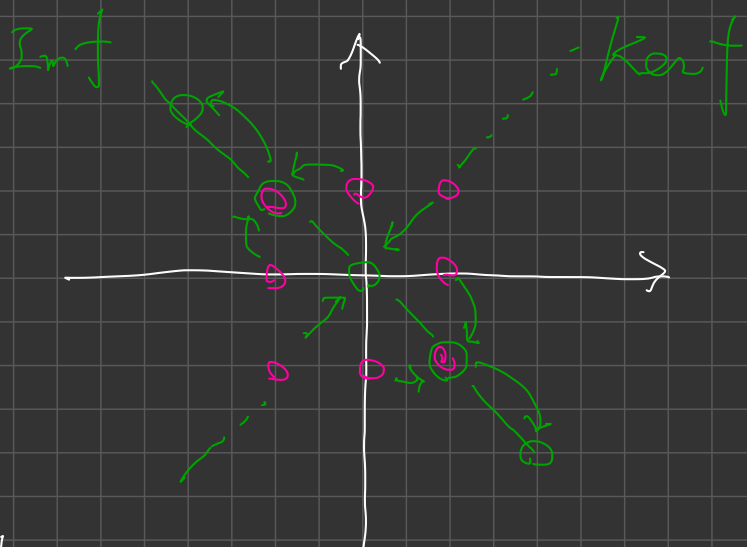
$$\text{Ker } f \quad 2 \text{ dim}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$



$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$|A| = 0 \quad \leftarrow \begin{matrix} \text{行列が0} \\ \text{逆行列が存在しない} \end{matrix}$$



$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1-1 \\ 1+1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1+1 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\begin{array}{r} 1 \quad -1 \\ -1 \quad 1 \\ \hline 1 \quad -1 \\ 0 \quad 0 \end{array}$$

$$x - y = 0$$

$$x = y$$

$$\text{Im}\{f\} = \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad y = -x$$

$$\text{Ker } f = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad y = x$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2

直交基底

$$\begin{array}{r}
 \begin{array}{ccc} 2 & -2 & -1 \\ 1 & -3 & 2 \end{array} \\
 \hline
 \begin{array}{ccc} 1 & -3 & 2 \\ 2 & -2 & -1 \end{array} \\
 \hline
 \begin{array}{ccc} 1 & -3 & 2 \\ 0 & 4 & -5 \end{array} \\
 \hline
 \begin{array}{ccc} 1 & -3 & 2 \\ 0 & 1 & -5/4 \end{array} \\
 \hline
 \begin{array}{ccc} 1 & 0 & -7/4 \\ 0 & 1 & -5/4 \end{array}
 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} +7 \\ +5 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} -7 \\ 0 \\ 1 \end{pmatrix}$$

$$V^2 \quad +7x_1 + 5x_2 + 4x_3 = 0$$

$$7x_1 + 5\alpha + 4\beta = 0$$

$$7x_1 = -5\alpha - 4\beta$$

$$x_1 = -\frac{5}{7}\alpha - \frac{4}{7}\beta$$

$$(2 \ -2 \ -1) \begin{pmatrix} +7 \\ +5 \\ 4 \end{pmatrix} = (14 \ -10 \ -4) = 0$$

$$(1 \ -3 \ 2) \begin{pmatrix} +7 \\ +5 \\ 4 \end{pmatrix} = 7 - 15 + 8 = 0$$

+②×3

$$\begin{array}{l}
 2 - 3 \frac{5}{4} \\
 = \frac{8-15}{4} \\
 = -\frac{7}{4}
 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -5/7 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -4/7 \\ 0 \\ 1 \end{pmatrix}$$

V^2 の基底

検算

3

$$\begin{array}{ccc|cc}
 a_1 & a_2 & a_3 & a & b \\
 1 & -1 & -1 & -2 & 2 \\
 -1 & -1 & -3 & 1 & -4 \\
 1 & -1 & -1 & 2 & 2 \\
 \hline
 1 & -1 & -1 & -2 & 2 \\
 0 & -2 & -4 & -1 & -2 \\
 0 & 0 & 0 & 4 & 0 \\
 \hline
 \end{array}$$

↑
OUT

$$\begin{array}{ccc|c}
 1 & -1 & -1 & 2 \\
 0 & 1 & -2 & 1 \\
 \hline
 1 & 0 & -3 & 3 \\
 0 & 1 & -2 & 1
 \end{array}$$

独立

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{aligned}
 x_2 - 2\lambda &= 1 \\
 x_2 &= 1 + 2\lambda
 \end{aligned}$$

$$\begin{aligned}
 x_1 - 3\lambda &= 3 \\
 x_1 &= 3 + 3\lambda
 \end{aligned}$$

$$\lambda = 0 \Rightarrow x_1 = 3, x_2 = 1, x_3 = 0$$

$$\boxed{b = 3a_1 + a_2}$$

$$3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$$

4

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & -2 & -1 \\ -3 & -\lambda & 1 \\ -3 & -2 & 3-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} -2-\lambda & -2-\lambda & 0 \\ -3 & -\lambda & 1 \\ -3 & -2 & 3-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} -2-\lambda & -2-\lambda & 0 \\ -3 & -\lambda & 1 \\ 0 & -2+\lambda & 2-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} -2-\lambda & 0 & 0 \\ -3 & -\lambda+3 & 1 \\ 0 & -2+\lambda & 2-\lambda \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & -2 & -1 \\ 0 & 2-\lambda & -2+\lambda \\ -3 & -2 & 3-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & -2 & -3 \\ 0 & 2-\lambda & 0 \\ -3 & -2 & 1-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & -2 & -3 \\ 0 & 2-\lambda & 0 \\ -4+\lambda & 0 & 4-\lambda \end{pmatrix}$$

$$= \begin{pmatrix} -2-\lambda & -2 & -3 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{pmatrix}$$

$-(4-\lambda)(2-\lambda)(2+\lambda)$

$\lambda = 4, 2, -2$

$\lambda = 4$

$$\begin{pmatrix} 1-4 & -2 & -1 \\ -3 & -4 & 1 \\ -3 & -2 & 3-4 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -2 & -1 \\ -3 & -4 & 1 \\ -3 & -2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -2 & -1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$-2y + 2z = 0$$

$$y = z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$-3x - 2\alpha - \alpha = 0$$

$$-3x = 3\alpha$$

$$x = \alpha$$

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix} \begin{pmatrix} -4 \\ 4 \\ 4 \end{pmatrix}$$

OK

$\lambda = 2$

$$\begin{pmatrix} 1-2 & -2 & -1 \\ -3 & -2 & 1 \\ -3 & -2 & 3-2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -1 \\ -3 & -2 & 1 \\ -3 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -1 \\ -3+3 & -2+6 & 1+3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -1 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$y = -z$$

$$-x + 2z - z = 0$$

$$-x + z = 0$$

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$$

OK

$\lambda = -2$

$$\begin{pmatrix} 1+2 & -2 & -1 \\ -3 & 2 & 1 \\ -3 & -2 & 3+2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -2 & -1 \\ -3 & 2 & 1 \\ -3 & -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -2 & -1 \\ 0 & -4 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$y = z$$

$$3x - 2z - z = 0$$

$$3x - 3z = 0$$

$$x = z$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ -3 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

OK

5

$$\begin{array}{cccc} 1 & 0 & -1 & 1 \\ 2 & -1 & -1 & 3 \\ -1 & 1 & 0 & -2 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 1 \\ 2-2 & -1 & -1+2 & 3-2 \\ 0 & 1 & -1 & -1 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}$$

Imf 2 dim $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

Kerf 2 dim

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$y - \alpha - \beta = 0$$

$$y = \alpha + \beta$$

$$x - \alpha + \beta = 0$$

$$x = \alpha - \beta$$