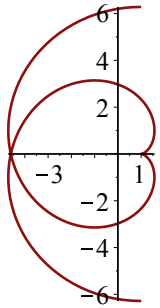
[1] diff & int

[1-(a)]パラメトリックプロットと微分

```
> x:=t->cos(t)+t*sin(t);
  y:=t->sin(t)-t*cos(t);
```

$$x := t \to \cos(t) + t\sin(t)$$
$$y := t \to \sin(t) - t\cos(t) \qquad \textbf{(1.1.1)}$$

```
> plot([x(t),y(t),t=- 2*Pi..2*Pi]);
```




```
> dx:=unapply(diff(x(t),t),t);
  dy:=unapply(diff(y(t),t),t);
```

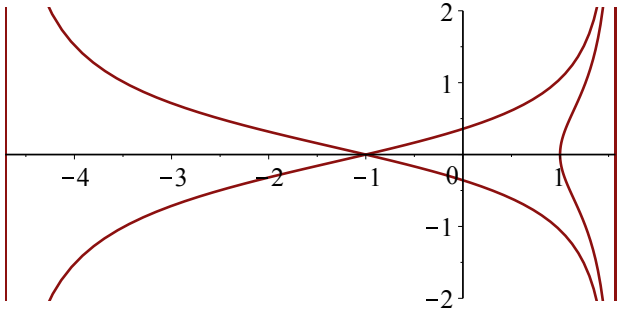
$$dx := t \to t\cos(t)$$
$$dy := t \to t\sin(t) \qquad \textbf{(1.1.2)}$$

```
> simplify(dy(t)/dx(t));
```

$$\frac{\sin(t)}{\cos(t)} \qquad \textbf{(1.1.3)}$$

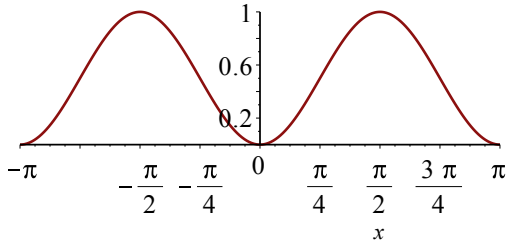
ちなみに，dy/dxをxについて表示するのは以下の通り．接線の傾きになっていることを読み取ってください．

```
> plot([x(t),dy(t)/dx(t),t=-1.5*Pi..1.5*Pi],view=-2..2);
```



[1-(b)]直交関数系の積分

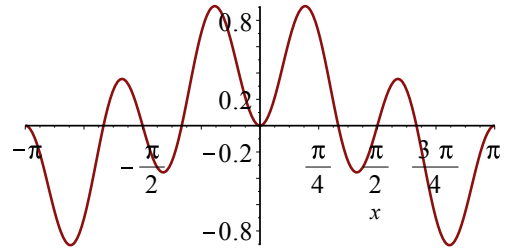
```
> plot(sin(x)*sin(x),x=-Pi..Pi);
```




```
> int(sin(x)*sin(x),x=-Pi..Pi);
```

$$\pi \qquad \textbf{(1.1.1.1)}$$

```
> plot(sin(2*x)*sin(3*x),x=-Pi..Pi);
```




```
> int(sin(2*x)*sin(3*x),x=-Pi..Pi);
```

$$0 \qquad \textbf{(1.1.1.2)}$$

n=mの時は全領域で関数値が正であるので，有限の値となるが，n<>mでは，同じ面積の図形が正負ペアで出現するため，積分を取るとキャンセルして0となる．

[2]LA

[2-(a)] 同時対角化

```
> restart:
  with(LinearAlgebra):
  A:=Matrix([[1,0,-2],[0,2,0],[-2,0,1]]);
  B:=Matrix([[3,0,2],[0,-3,0],[2,0,3]]);
```

$$A := \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$B := \begin{bmatrix} 3 & 0 & 2 \\ 0 & -3 & 0 \\ 2 & 0 & 3 \end{bmatrix} \quad (2.1.1)$$

```
> A.B;
```

$$\begin{bmatrix} -1 & 0 & -4 \\ 0 & -6 & 0 \\ -4 & 0 & -1 \end{bmatrix} \quad (2.1.2)$$

```
> B.A;
```

$$\begin{bmatrix} -1 & 0 & -4 \\ 0 & -6 & 0 \\ -4 & 0 & -1 \end{bmatrix} \quad (2.1.3)$$

```
> Eigenvectors(A);
```

$$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad (2.1.4)$$

```
> v,P:=Eigenvectors(B);
```

$$v, P := \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad (2.1.5)$$

```
> Transpose(P).A.P;
```

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (2.1.6)$$

```
> Transpose(P).B.P;
```

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 10 \end{bmatrix} \quad (2.1.7)$$

[2-(b)] 2次形式

```
> restart:
  with(LinearAlgebra):
  #A:=Matrix([[1,1,-1],[1,2,1],[-1,1,3]]);
  A:=Matrix([[3,2,2],[2,2,0],[2,0,4]]);
```

$$A := \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix} \quad (2.2.1)$$

```
> l,P:=Eigenvectors(A):
> x:=Vector([x1,x2,x3]);
```

$$x := \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \quad (2.2.2)$$

```
> tx:=Transpose(x);
```

$$tx := \begin{bmatrix} x1 & x2 & x3 \end{bmatrix} \quad (2.2.3)$$

```
> expand(tx.A.x);
```

$$3\,x1^2 + 4\,x1\,x2 + 4\,x1\,x3 + 2\,x2^2 + 4\,x3^2 \quad (2.2.4)$$

```
> #P:=Matrix([[1,-1,3],[0,1,-2],[0,0,1]]);
  P:=<Normalize(Column(P,1),Euclidean)|
  Normalize(Column(P,2),Euclidean)|
  Normalize(Column(P,3),Euclidean)>;
  y:=Vector([y1,y2,y3]);
```

$$P := \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$y := \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} \quad (2.2.5)$$

```
> tPy:=Transpose(P.y);
```

$$tPy := \quad (2.2.6)$$

$$\begin{bmatrix} -\frac{2}{3}\,y1 - \frac{1}{3}\,y2 + \frac{2}{3}\,y3 & \frac{2}{3}\,y1 - \frac{2}{3}\,y2 + \frac{1}{3}\,y3 & \frac{1}{3}\,y1 + \frac{2}{3}\,y2 + \frac{2}{3}\,y3 \end{bmatrix}$$

```
> expand(tPy.A.P.y);
```

$$3\,y2^2 + 6\,y3^2 \quad (2.2.7)$$

[3]

[3-(a)]

```
> restart;
```

```
a_b_c:=1;
ab_bc_ca:=-2;
abc:=-1;
```

$$a\_b\_c := 1$$
$$ab\_bc\_ca := -2$$
$$abc := -1 \qquad \textbf{(3.1.1)}$$

ア

```
> expand((a+b+c)^2-2*(a*b+a*c+b*c));
  a2_b2_c2:=a_b_c^2-2*(ab_bc_ca);
```

$$a^2 + b^2 + c^2$$
$$a2\_b2\_c2 := 5 \qquad \textbf{(3.1.2)}$$

イ

```
> normal(1/a+1/b+1/c);
  a1_b1_c1:=ab_bc_ca/abc;
```

$$\frac{a\,b + a\,c + b\,c}{a\,b\,c}$$
$$a1\_b1\_c1 := 2 \qquad \textbf{(3.1.3)}$$

```
> normal(expand((1/a+1/b+1/c)^2-(1/a^2+1/b^2+1/c^2)));
```

$$\frac{2\,(a + b + c)}{a\,b\,c} \qquad \textbf{(3.1.4)}$$

ウ

```
> a21_b21_c21:=solve(2*a_b_c/abc=a1_b1_c1^2-X,X);
```

$$a21\_b21\_c21 := 6 \qquad \textbf{(3.1.5)}$$

```
> A:=(a*x-1/a)^2+(b*x-1/b)^2+(c*x-1/c)^2
```

$$A := \left(a\,x - \frac{1}{a}\right)^2 + \left(b\,x - \frac{1}{b}\right)^2 + \left(c\,x - \frac{1}{c}\right)^2 \qquad \textbf{(3.1.6)}$$

```
> collect(expand(A),x);
```

$$\left(a^2 + b^2 + c^2\right) x^2 - 6\,x + \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \qquad \textbf{(3.1.7)}$$

エ，オ，カ

```
> eq:=a2_b2_c2*x^2-6*x+a21_b21_c21;
```

$$eq := 5\,x^2 - 6\,x + 6 \qquad \textbf{(3.1.8)}$$

キ，クケ，コ

```
> solve(7=eq,x);
```

$$\frac{3}{5} - \frac{1}{5}\sqrt{14},\ \frac{3}{5} + \frac{1}{5}\sqrt{14} \qquad \textbf{(3.1.9)}$$

[3-(b)]

```
> a_b_c:=1.1;
  ab_bc_ca:=-2.2;
  abc:=-1.1;
```

$$a\_b\_c := 1.1$$
$$ab\_bc\_ca := -2.2$$
$$abc := -1.1 \qquad \textbf{(3.2.1)}$$

ア

```
> expand((a+b+c)^2-2*(a*b+a*c+b*c));
  a2_b2_c2:=a_b_c^2-2*(ab_bc_ca);
```

$$a^2 + b^2 + c^2$$
$$a2\_b2\_c2 := 5.61 \qquad \textbf{(3.2.2)}$$

イ

```
> normal(1/a+1/b+1/c);
  a1_b1_c1:=ab_bc_ca/abc;
```

$$\frac{a\,b + a\,c + b\,c}{a\,b\,c}$$
$$a1\_b1\_c1 := 2.000000000 \qquad \textbf{(3.2.3)}$$

```
> normal(expand((1/a+1/b+1/c)^2-(1/a^2+1/b^2+1/c^2)));
```

$$\frac{2\,(a + b + c)}{a\,b\,c} \qquad \textbf{(3.2.4)}$$

ウ

```
> a21_b21_c21:=solve(2*a_b_c/abc=a1_b1_c1^2-X,X);
```

$$a21\_b21\_c21 := 6. \qquad \textbf{(3.2.5)}$$

```
> A:=(a*x-1/a)^2+(b*x-1/b)^2+(c*x-1/c)^2
```

$$A := \left(a\,x - \frac{1}{a}\right)^2 + \left(b\,x - \frac{1}{b}\right)^2 + \left(c\,x - \frac{1}{c}\right)^2 \qquad \textbf{(3.2.6)}$$

```
> collect(expand(A),x);
```

$$\left(a^2 + b^2 + c^2\right) x^2 - 6\,x + \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \qquad \textbf{(3.2.7)}$$

エ，オ，カ

```
> eq:=a2_b2_c2*x^2-6*x+a21_b21_c21;
```

$$eq := 5.61\,x^2 - 6\,x + 6. \qquad \textbf{(3.2.8)}$$

キ，クケ，コ

```
> solve(7=eq,x);
```

$$-0.1465780696,\ 1.216096786 \qquad \textbf{(3.2.9)}$$