1

```
> restart;
  diff(log(1+x),x);
```

$$\frac{1}{1+x} \tag{1.1}$$

```
> restart;
  eq1:=log((1+x)/(1-x));
  eq2:=series(eq1,x);
```

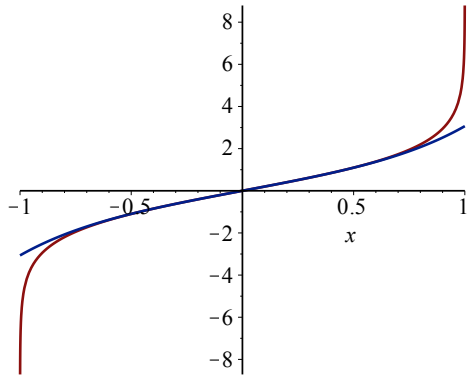
$$eq1 := \ln\left(\frac{1+x}{1-x}\right)$$

$$eq2 := 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + O\left(x^7\right) \tag{1.2}$$

```
> eq3:=convert(eq2,polynom);
```

$$eq3 := 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 \tag{1.3}$$

```
> plot([eq1,eq3],x=-1..1);
```



2

(a)

```
> restart;
  eq1:=sin(x)^2/(a^2*sin(x)^2+b^2*cos(x)^2);
```

$$eq1 := \frac{\sin(x)^2}{a^2\sin(x)^2 + b^2\cos(x)^2} \tag{2.1.1}$$

```
> int(eq1,x=0..Pi/2);
```

$$-\frac{1}{2}\,\frac{\pi\left(b\,\mathrm{csgn}\left(\frac{a}{b}\right) - a\right)}{\left(a^2 - b^2\right)a} \tag{2.1.2}$$

```
> assume(b>0);
```

```
> assume(a>0);
> int(eq1,x=0..Pi/2);
```

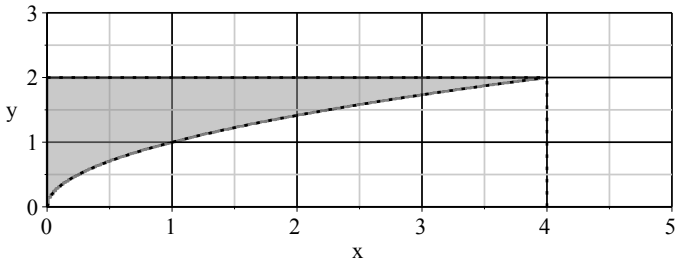
$$\frac{1}{2}\,\frac{\pi}{a\sim\,(a\sim + b\sim)} \tag{2.1.3}$$

(b)

```
> int(int(1/sqrt(y^3+1),y=sqrt(x)..2),x=0..4);
```

$$\int_0^4 \left(-\frac{1}{3}\,\frac{1}{\sqrt{x^{3/2}+1}} \left(\mathrm{I}\sqrt{3} - 3\right) \left(\mathrm{EllipticF}\left(\sqrt{3}\,\sqrt{-\frac{2}{\mathrm{I}\sqrt{3}-3}}, \sqrt{-\frac{\mathrm{I}\sqrt{3}-3}{\mathrm{I}\sqrt{3}+3}} \right) \sqrt{\frac{\mathrm{I}\sqrt{3}+3}{\mathrm{I}\sqrt{3}-3}}\,\sqrt{\frac{\mathrm{I}\sqrt{3}-3}{\mathrm{I}\sqrt{3}+3}}\,\sqrt{3}\,\sqrt{-\frac{2}{\mathrm{I}\sqrt{3}-3}}\,\sqrt{x^{3/2}+1} - 3\,\mathrm{EllipticF}\left(\sqrt{-\frac{2\sqrt{x}+2}{\mathrm{I}\sqrt{3}-3}}, \sqrt{-\frac{\mathrm{I}\sqrt{3}-3}{\mathrm{I}\sqrt{3}+3}} \right) \sqrt{\frac{\mathrm{I}\sqrt{3}+2\sqrt{x}-1}{\mathrm{I}\sqrt{3}-3}}\,\sqrt{\frac{\mathrm{I}\sqrt{3}-2\sqrt{x}+1}{\mathrm{I}\sqrt{3}+3}}\,\sqrt{-\frac{2\sqrt{x}+2}{\mathrm{I}\sqrt{3}-3}} \right) \right) dx \tag{2.2.1}$$

```
> with(plots):
  inequal({y-sqrt(x)>0,y<2,x<4},x=0..5,y=0..3);
```




```
> ?inequal;
> int(int(1/sqrt(y^3+1),x=0..y^2),y=0..2);
```

$$\frac{4}{3} \tag{2.2.2}$$

3

(a)

```
> A:=Matrix([[1,-2,-2],[2,-3,-2],[-2,2,1]]);
```

$$A := \begin{bmatrix} 1 & -2 & -2 \\ 2 & -3 & -2 \\ -2 & 2 & 1 \end{bmatrix} \quad (3.1.1)$$

```
> with(LinearAlgebra):
> l,P:=Eigenvectors(A);
```

$$l, P := \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (3.1.2)$$

```
> MatrixInverse(P).A.P;
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (3.1.3)$$

(b)

```
> A:=Matrix([[0,-1],[1,1]]);
```

$$A := \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \quad (3.2.1)$$

```
> E:=IdentityMatrix(2);
```

$$E := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.2.2)$$

```
> f:=unapply(Determinant(A-t*E),t);
```

$$f := t \to t^2 - t + 1 \quad (3.2.3)$$

```
> f(A);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.2.4)$$

4

original

```
> restart;
> eq1:=2*x^2-4*(a+1)*x+10*a+1;
```

$$eq1 := 2x^2 - 4(a+1)x + 10a + 1 \quad (4.1.1)$$

```
> sol1:=solve(diff(eq1,x),x);
```

$$sol1 := a + 1 \quad (4.1.2)$$

```
> eq2:=expand(subs(x=sol1,eq1));
```

$$eq2 := -2a^2 + 6a - 1 \quad (4.1.3)$$

```
> solve(eq2,a);
```

$$\frac{3}{2} - \frac{1}{2}\sqrt{7}, \frac{3}{2} + \frac{1}{2}\sqrt{7} \quad (4.1.4)$$

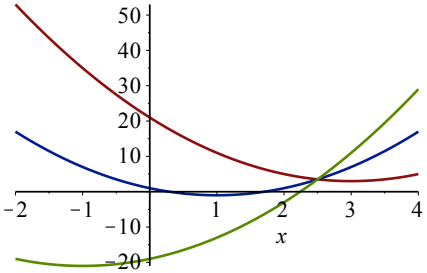
```
> a1:=solve(sol1=-1,a);
```

$$a1 := -2 \quad (4.1.5)$$

```
> a2:=solve(sol1=3,a);
```

$$a2 := 2 \quad (4.1.6)$$

```
> plot([subs(a=a2,eq1),subs(a=0,eq1),subs(a=a1,eq1)],
  x=-2..4);
```




```
> eq3:=subs(x=-1,eq1);
```

$$eq3 := 7 + 14a \quad (4.1.7)$$

```
> eq4:=subs(x=3,eq1);
```

$$eq4 := 7 - 2a \quad (4.1.8)$$

```
> solve(eq2=7/9,a);
```

$$\frac{1}{3}, \frac{8}{3} \quad (4.1.9)$$

```
> solve(eq4=7/9,a);
```

$$\frac{28}{9} \quad (4.1.10)$$

modified

```
> restart;
> eq1:=2*x^2-4.2*(a+1)*x+10*a+1;
```

$$eq1 := 2x^2 - 4.2(a+1)x + 10a + 1 \quad (4.2.1)$$

```
> sol1:=solve(diff(eq1,x),x);
```

$$sol1 := 1.050000000a + 1.050000000 \quad (4.2.2)$$

```
> eq2:=expand(subs(x=sol1,eq1));
```

$$eq2 := -2.205000000a^2 + 5.590000000a - 1.205000000 \quad (4.2.3)$$

```
> solve(eq2,a);
```

$$0.2378854839, 2.297261908 \quad (4.2.4)$$

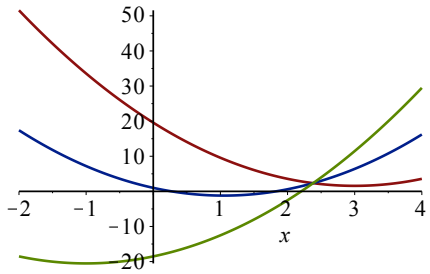
```
> a1:=solve(sol1=-1,a);
```

$$a1 := -1.952380952 \quad (4.2.5)$$

```
> a2:=solve(sol1=3,a);
```

$$a2 := 1.857142857 \quad (4.2.6)$$

```
> plot([subs(a=a2,eq1),subs(a=0,eq1),subs(a=a1,eq1)],
  x=-2..4);
```




```
> eq3:=subs(x=-1,eq1);
```

$$eq3 := 7.2 + 14.2\,a \qquad \textbf{(4.2.7)}$$

```
> eq4:=subs(x=3,eq1);
```

$$eq4 := 6.4 - 2.6\,a \qquad \textbf{(4.2.8)}$$

```
> #m0:=7/9;
  m0:=0.8;
  sol2:=solve(eq2=m0,a);
```

$$m0 := 0.8$$

$$sol2 := 0.4324413575,\ 2.102706035 \qquad \textbf{(4.2.9)}$$

```
> evalf(sol2);
```

$$0.4324413575,\ 2.102706035 \qquad \textbf{(4.2.10)}$$

```
> sol3:=solve(eq4=m0,a);
```

$$sol3 := 2.153846154 \qquad \textbf{(4.2.11)}$$

```
> plot([subs(a=sol2[1],eq1),subs(a=sol2[2],eq1),subs
  (a=sol3,eq1)],x=-2..4,y=0.7..0.9);
```