1a

```
> int(int(sqrt(x),y=-sqrt(x-x^2)..sqrt(x-x^2)),x=0..1);
```

$$\frac{8}{15} \tag{1.1}$$

```
> x/(x^2+y^2);
```

$$\frac{x}{x^2+y^2} \tag{1.2}$$

```
> int(int(x/(x^2+y^2),y=-sqrt(x-x^2)..sqrt(x-x^2)),x=0.
  .1);
```

$$\frac{1}{2}\pi \tag{1.3}$$

1b

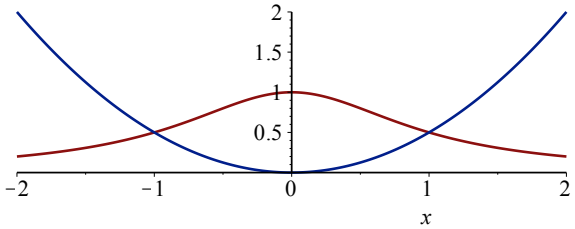
```
> y1:=x->1/(x^2+1);
```

$$y1 := x \to \frac{1}{x^2+1} \tag{2.1}$$

```
> y2:=x->x^2/2;
```

$$y2 := x \to \frac{1}{2}x^2 \tag{2.2}$$

```
> plot([y1(x),y2(x)],x=-2..2);
```




```
> solve(y1(x)=y2(x),x);
```

$$1, -1, I\sqrt{2}, -I\sqrt{2} \tag{2.3}$$

```
> int(y1(x)-y2(x),x=-1..1);
```

$$\frac{1}{2}\pi - \frac{1}{3} \tag{2.4}$$

2a

```
> A:=Matrix([[5,-7,-7],[-4,8,7],[4,-10,-9]]);
```

$$A := \begin{bmatrix} 5 & -7 & -7 \\ -4 & 8 & 7 \\ 4 & -10 & -9 \end{bmatrix} \tag{3.1}$$

```
> l,P:=Eigenvectors(A);
```

$$l, P := \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \frac{1}{2} \\ -1 & -1 & -\frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix} \tag{3.2}$$

```
> MatrixInverse(P).A.P;
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{3.3}$$

2b

```
> restart;
> with(LinearAlgebra):
> Q:=Matrix([[0.6,0.2,0.2,0.2],[0.2,0.6,0.2,0.1],[0.1,
  0.1,0.5,0.2],[0.1,0.1,0.1,0.5]]);
```

$$Q := \begin{bmatrix} 0.6 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.5 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.5 \end{bmatrix} \tag{4.1}$$

```
> xx:=Vector([60,40,0,0]);
```

$$xx := \begin{bmatrix} 60 \\ 40 \\ 0 \\ 0 \end{bmatrix} \tag{4.2}$$

```
> Q.xx;
```

$$\begin{bmatrix} 44. \\ 36. \\ 10. \\ 10. \end{bmatrix} \tag{4.3}$$

```
> (Q^(10)).xx;
```

$$\begin{bmatrix} 33.3361295360000 \\ 30.5609149440000 \\ 19.4380364800000 \\ 16.6649190400000 \end{bmatrix} \tag{4.4}$$

3a

```
> restart;
  P:=[[2,0],[4,0],[6,0]];
```

(5.1)

$$P := [[2, 0], [4, 0], [6, 0]] \quad (5.1)$$

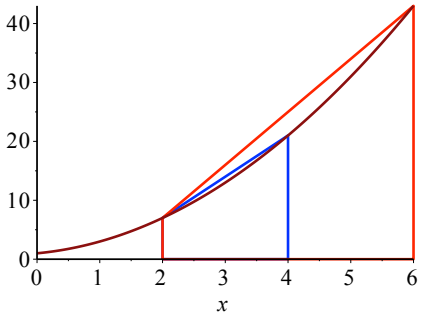
```
> f:=unapply(subs([a=1,b=1,c=1],a*x^2+b*x+c),x);
```

$$f := x \to x^2 + x + 1 \quad (5.2)$$

```
> with(plots):
  l1:=listplot([P[1],P[2],[4,f(4)],[2,f(2)],P[1]],color=
  blue);
  l2:=listplot([P[],P[3],[6,f(6)],[2,f(2)],P[1]],color=
  red);
  fg:=plot(f(x),x=0..6):
```

$$l1 := PLOT(\ldots)$$
$$l2 := PLOT(\ldots) \quad (5.3)$$

```
> display(l1,l2,fg);
```




```
> G:=unapply(a*x^2+b*x+c,x);
```

$$G := x \to a\,x^2 + b\,x + c \quad (5.4)$$

```
> S1:=2*(G(4)+G(2))/2;
```

$$S1 := 20\,a + 6\,b + 2\,c \quad (5.5)$$

```
> S2:=4*(G(6)+G(2))/2;
```

$$S2 := 80\,a + 16\,b + 4\,c \quad (5.6)$$

```
> eq1:=S2-(S1+2*(G(6)+G(4))/2);
```

$$eq1 := 8\,a \quad (5.7)$$

```
> solve(eq1=16,a);
```

$$2 \quad (5.8)$$

3b

```
> s1:=solve(subs(a=2,G(-2)=2),c);
```

$$s1 := -6 + 2\,b \quad (6.1)$$

```
> G1:=unapply(subs([a=2,c=s1],G(x)),x);
```

$$G1 := x \to 2\,x^2 + b\,x - 6 + 2\,b \quad (6.2)$$

```
> s2:=solve(diff(G1(x),x)=0,x);
```

$$s2 := -\frac{1}{4}\,b \quad (6.3)$$

```
> G1(s2);
```

$$-\frac{1}{8}\,b^2 - 6 + 2\,b \quad (6.4)$$

```
> solve(G1(s2)>0,b);
```

$$RealRange(Open(4), Open(12)) \quad (6.5)$$

```
> H:=unapply(expand(2*(x+2)^2)+k,x);
```

$$H := x \to 2\,x^2 + 8\,x + 8 + k \quad (6.6)$$

```
> G1(x);
```

$$2\,x^2 + b\,x - 6 + 2\,b \quad (6.7)$$

```
> collect(G1(x)-H(x),x);
```

$$(b - 8)\,x - 14 + 2\,b - k \quad (6.8)$$

```
> subs(b=8,coeff(G1(x)-H(x),x,0));
```

$$2 - k \quad (6.9)$$

```
> solve(subs(b=8,coeff(G1(x)-H(x),x,0)=0),k);
```

$$2 \quad (6.10)$$

```
> subs(a=2,b=8,G(x));
```

$$2\,x^2 + 8\,x + c \quad (6.11)$$

```
> subs(k=2,H(x));
```

$$2\,x^2 + 8\,x + 10 \quad (6.12)$$