

10

1(a)

```
> restart;
diff(sqrt((x^2-1)/(x^2+1)),x);
```

$$\frac{1}{2} \frac{\frac{2x}{x^2+1} - \frac{2(x^2-1)x}{(x^2+1)^2}}{\sqrt{\frac{x^2-1}{x^2+1}}} \quad (1.1)$$

10

15

1(b)

```
> assume(x>0);
I1:=int(log(x/y^2),y=1..x);
I1 := -ln(x) - 2 - ln(x) x + 2 x
> int(I1,x=1..2);
```

$$\frac{11}{4} - 4 \ln(2) \quad (2.1)$$

15

10

2(a)

```
> restart;
with(LinearAlgebra);
> A:=Matrix([[1,2,3],[2,1,3],[3,3,2]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 2 \end{bmatrix} \quad (3.1)$$

```
> E:=IdentityMatrix(3);
```

$$E := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.2)$$

```
> Determinant(A-t*E);
```

$$12 + 17t + 4t^2 - t^3 \quad (3.3)$$

```
> solve(Determinant(A-t*E)=0,t);
```

$$\left[-1, \frac{5}{2} - \frac{1}{2}\sqrt{73}, \frac{5}{2} + \frac{1}{2}\sqrt{73} \right] \quad (3.4)$$

10

```
> Eigenvectors(A);
```

(3.5)

$$\begin{bmatrix} \frac{5}{2} + \frac{1}{2}\sqrt{73} \\ \frac{5}{2} - \frac{1}{2}\sqrt{73} \\ -1 \end{bmatrix} \quad \text{5 部分}$$

$$\begin{bmatrix} 4 \left(\frac{7}{2} + \frac{1}{2}\sqrt{73} \right) \\ \frac{4 \left(\frac{7}{2} + \frac{1}{2}\sqrt{73} \right)}{\left(\frac{5}{2} + \frac{1}{2}\sqrt{73} \right) \left(\frac{3}{2} + \frac{1}{2}\sqrt{73} \right)}, \\ \frac{4 \left(\frac{7}{2} - \frac{1}{2}\sqrt{73} \right)}{\left(\frac{5}{2} - \frac{1}{2}\sqrt{73} \right) \left(\frac{3}{2} - \frac{1}{2}\sqrt{73} \right)}, -1 \end{bmatrix}$$

$$\left[\frac{1}{2} \frac{\frac{13}{2} + \frac{1}{2}\sqrt{73}}{\frac{5}{2} + \frac{1}{2}\sqrt{73}}, \frac{1}{2} \frac{\frac{13}{2} - \frac{1}{2}\sqrt{73}}{\frac{5}{2} - \frac{1}{2}\sqrt{73}}, 1 \right]$$

$$\begin{bmatrix} 1, 1, 0 \end{bmatrix}$$

(3.5)

15

2(b)

```
> restart;
with(LinearAlgebra);
> Q:=Matrix([[3,2],[2,6]]);
xx:=Vector([x,y]);
bb:=Vector([-6,-2]);
```

$$Q := \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} \quad (4.1)$$

$$xx := \begin{bmatrix} x \\ y \end{bmatrix}$$

$$bb := \begin{bmatrix} -6 \\ -2 \end{bmatrix} \quad (4.1)$$

```
> expand(Transpose(xx).Q.xx+Transpose(bb).xx+2);
```

$$3x^2 + 4xy + 6y^2 - 6x - 2y + 2 \quad (4.2)$$

```
> I,V:=Eigenvectors(Q);
```

(4.3)

$$V := \begin{bmatrix} 2 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 & \frac{1}{2} \\ 1 & 1 \end{bmatrix}$$

4/5 部分

```
> v1:=Normalize(Column(V,1),Euclidean);
v2:=Normalize(Column(V,2),Euclidean);
```

$$v1 := \begin{bmatrix} -\frac{2}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} \end{bmatrix}$$

$$v2 := \begin{bmatrix} \frac{1}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} \end{bmatrix}$$

(4.4)

```
> P:=<v1|v2>;
```

$$P := \begin{bmatrix} -\frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} & \frac{2}{5}\sqrt{5} \end{bmatrix}$$

(4.5)

```
> Transpose(P).Q.P;
```

$$\begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}$$

(4.6)

```
> yy:=Vector([xp,yp]);
```

$$yy := \begin{bmatrix} xp \\ yp \end{bmatrix}$$

(4.7)

```
> e1:=expand(Transpose(yy).Transpose(P).Q.P.yy+Transpose(bb).P.yy+2;
```

$$e1 := 2xp^2 + 7yp^2 + 2xp\sqrt{5} - 2yp\sqrt{5} + 2$$

(4.8)

```
> expand(7*(xp-sqrt(5)/7)^2+2*(yp+sqrt(5)/2)^2+2-45/14);
```

$$7xp^2 - 2xp\sqrt{5} + 2 + 2yp^2 + 2yp\sqrt{5}$$

9/15 (4.9)

3(a)

```
> restart;
```

$$eq1 := (2^x + 3^x) * (9/2^x + 4/3^x);$$

$$eq1 := (2^x + 3^x) \left(\frac{9}{2^x} + \frac{4}{3^x} \right)$$

(5.1)

```
> expand(eq1);
```

(5.2)

$$13 + \frac{4 \cdot 2^x}{3^x} + \frac{9 \cdot 3^x}{2^x}$$

```
> expand(13+4/X+9*X);
```

$$13 + \frac{4}{X} + 9X$$

13

```
> eq2:=13+4/X+9*X-a;
```

$$eq2 := 13 + \frac{4}{X} + 9X - a$$

(5.2)

(5.3)

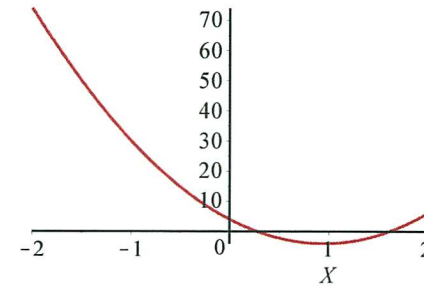
```
> sort(expand(eq2*X),X);
```

$$9X^2 + 13X - aX + 4$$

(5.4)

(5.5)

```
> plot(subs(a=30,eq2*X),X=-2..2);
```



```
> s1:=solve(expand(eq2*X),X);
```

$$s1 := -\frac{13}{18} + \frac{1}{18}a + \frac{1}{18}\sqrt{25 - 26a + a^2}, -\frac{13}{18} + \frac{1}{18}a - \frac{1}{18}\sqrt{25 - 26a + a^2}$$

(5.6)

```
> solve(-13/18+a/18>0,a);
```

$$\text{RealRange}(\text{Open}(13), \infty)$$

(5.7)

```
> solve(25-26*a+a^2>0,a);
```

$$\text{RealRange}(-\infty, \text{Open}(1)), \text{RealRange}(\text{Open}(25), \infty)$$

(5.8)

```
> a>25;
```

$$25 < a$$

5

(5.9)

```
> expand(s1[1]*s1[2]);
```

$$\frac{4}{9}$$

(5.10)

```
> 4/9=(3/2)^(x1+x2);
```

$$\frac{4}{9} = \left(\frac{3}{2} \right)^{x1+x2}$$

(5.11)

```
> simplify(solve(4/9=(3/2)^(x3),x3));
```

LL

$$-2 \rightarrow (5) \quad (5.12)$$

25

3(b)

```
> s2:=solve(subs(a=50,eq2),X);
```

$$s2 := 4, \frac{1}{9} \rightarrow (10) \quad (6.1)$$

```
> expand(solve(s2[1]=(3/2)^x,x));
```

$$\frac{2 \ln(2)}{\ln(3) - \ln(2)} \rightarrow (5) \quad (6.2)$$

```
> expand(solve(s2[2]=(3/2)^x,x));
```

$$-\frac{2 \ln(3)}{\ln(3) - \ln(2)} \rightarrow (5) \quad (6.3)$$

$$\frac{(2)}{\log_2 3 - (1)}$$

$$\frac{(-2)}{\log_2 3 - (1)}$$

$$\rightarrow (10)$$