

1(a)

```
> e1:=int(sqrt(x),y=-sqrt(x-x^2)..sqrt(x-x^2));
```

$$e1 := 2\sqrt{x}\sqrt{x-x^2}$$

(1.1)

```
> int(e1,x=0..1);
```

$$\frac{8}{15}$$

(1.2)

1(b)

```
> x1:=a*cos(t)^3;
```

$$x1 := a \cos(t)^3$$

(2.1)

```
> y1:=a*sin(t)^3;
```

$$y1 := a \sin(t)^3$$

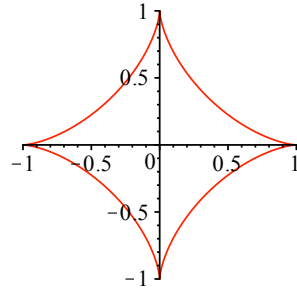
(2.2)

```
> diff(y1,t)/diff(x1,t);
```

$$-\frac{\sin(t)}{\cos(t)}$$

(2.3)

```
> plot([cos(t)^3,sin(t)^3,t=0..2*Pi]);
```



2(1)

```
> with(LinearAlgebra):
```

```
> A:=Matrix([[1,-1,1],[-7,2,1],[2,1,2]]);
```

$$A := \begin{bmatrix} 1 & -1 & 1 \\ -7 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

(3.1)

```
> l,P:=Eigenvectors(A);
```

$$l,P := \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 & 0 & -1 \\ 4 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

(3.2)

```
> MatrixInverse(P).A.P;
```

(3.3)

2(b)

```
> with(LinearAlgebra):
```

```
> a1:=Vector([-1,0,2]);
```

```
    a2:=Vector([0,1,1]);
```

```
    a3:=Vector([3,-1,0]);
```

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

(3.3)

$$a1 := \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$$a2 := \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$a3 := \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

(4.1)

```
> b1:=Vector([-5,0,3]);
```

```
    b2:=Vector([0,1,6]);
```

```
    b3:=Vector([-5,-1,9]);
```

$$b1 := \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}$$

$$b2 := \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix}$$

$$b3 := \begin{bmatrix} -5 \\ -1 \\ 9 \end{bmatrix}$$

(4.2)

```
> A:=<b1|b2|b3>.MatrixInverse(<a1|a2|a3>);
```

$$A := \begin{bmatrix} -\frac{5}{7} & \frac{20}{7} & -\frac{20}{7} \\ 0 & 1 & 0 \\ \frac{27}{7} & \frac{18}{7} & \frac{24}{7} \end{bmatrix}$$

(4.3)

```
> A.a3;
```

(4.4)

$$\begin{bmatrix} -5 \\ -1 \\ 9 \end{bmatrix}$$

3&4

```
> restart;
C1:=unapply(x^2,x);
```

$$C1 := x \rightarrow x^2$$

```
> m1:=unapply(diff(C1(x),x),x);
```

$$m1 := x \rightarrow 2x$$

```
> l:=unapply(expand(m1(t)*(x-t)+t^2),x);
```

$$l := x \rightarrow 2tx - t^2$$

```
> C2:=unapply(x^2-4*a*x+4*a*(a+1),x);
```

$$C2 := x \rightarrow x^2 - 4ax + 4a(a+1)$$

```
> e0:=collect(C2(x)-l(x),x);
```

$$e0 := x^2 + (-4a - 2t)x + t^2 + 4a(a+1)$$

```
> b1:=coeff(e0,x);
c1:=coeff(e0,x,0);
e1:=expand(b1^2-4*c1);
solve(e1=0,t);
```

$$b1 := -4a - 2t$$

$$c1 := t^2 + 4a(a+1)$$

$$e1 := 16at - 16a$$

$$1$$

```
> t:=1;
```

$$t := 1$$

```
> expand(C2(x)-l(x));
solve(C2(x)-l(x)=0,x);
```

$$x^2 - 4ax + 4a^2 + 4a - 2x + 1$$

$$2a+1, 2a+1$$

```
> expand(C2(2*a+1));
```

$$1 + 4a$$

```
> s2:=subs(a=2,[C1(x),C2(x),l(x)]);
```

$$s2 := [x^2, x^2 - 8x + 24, 2x - 1]$$

```
> plot(s2,x,y=-10..20);
```

(4.4)

(5.1)

(5.2)

(5.3)

(5.4)

(5.5)

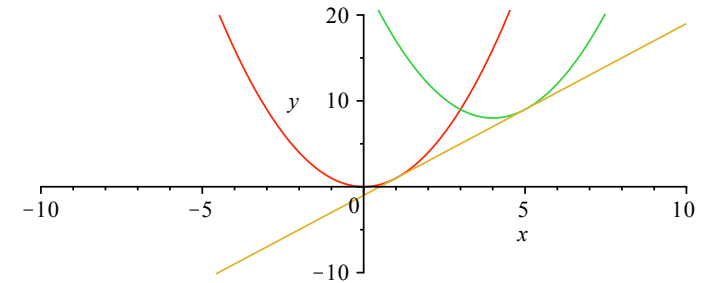
(5.6)

(5.7)

(5.8)

(5.9)

(5.10)



```
> solve(C1(x)=C2(x),x);
```

$$a+1$$

(5.11)

```
> l2:=unapply(expand(m1(1)*(x-(a+1))+C1(a+1)),x);
```

$$l2 := x \rightarrow 2x - 1 + a^2$$

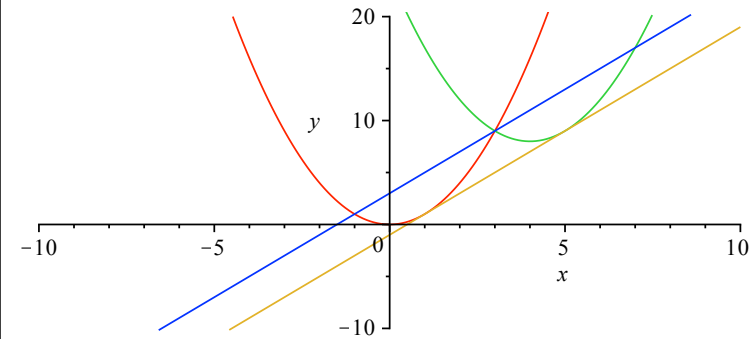
(5.12)

```
> s3:=subs(a=2,[C1(x),C2(x),l(x),l2(x)]);
```

$$s3 := [x^2, x^2 - 8x + 24, 2x - 1, 2x + 3]$$

(5.13)

```
> plot(s3,x,y=-10..20);
```



```
> factor(expand(int(l2(x)-C1(x),x=0..a+1)));
```

$$\frac{1}{3}(2a-1)(a+1)^2$$

(5.14)