

1(a)

```
> restart;
diff(sqrt((x^2-1)/(x^2+1)),x);
```

$$\frac{1}{2} \frac{\frac{2x}{x^2+1} - \frac{2(x^2-1)x}{(x^2+1)^2}}{\sqrt{\frac{x^2-1}{x^2+1}}} \quad (1.1)$$

1(b)

```
> assume(x>0);
I1:=int(log(x/y^2),y=1..x);
```

$$I1 := -\ln(x) - 2 - \ln(x) x + 2x \quad (2.1)$$

```
> int(I1,x=1..2);
```

$$\frac{11}{4} - 4 \ln(2) \quad (2.2)$$

2(a)

```
> restart;
with(LinearAlgebra);
> A:=Matrix([[1,2,3],[2,1,3],[3,3,2]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 2 \end{bmatrix} \quad (3.1)$$

```
> E:=IdentityMatrix(3);
```

$$E := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.2)$$

```
> Determinant(A-t*E);
```

$$12 + 17t + 4t^2 - t^3 \quad (3.3)$$

```
> solve(Determinant(A-t*E)=0,t);
```

$$-1, \frac{5}{2} - \frac{1}{2}\sqrt{73}, \frac{5}{2} + \frac{1}{2}\sqrt{73} \quad (3.4)$$

2(b)

```
> restart;
```

```
with(LinearAlgebra);
> Q:=Matrix([[3,2],[2,6]]);
xx:=Vector([x,y]);
bb:=Vector([-6,-2]);
```

$$Q := \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} \quad (4.1)$$

$$xx := \begin{bmatrix} x \\ y \end{bmatrix}$$

$$bb := \begin{bmatrix} -6 \\ -2 \end{bmatrix}$$

```
> expand(Transpose(xx).Q.xx+Transpose(bb).xx+2);
```

$$3x^2 + 4xy + 6y^2 - 6x - 2y + 2 \quad (4.2)$$

```
> l,V:=Eigenvectors(Q);
```

$$l, V := \left[\begin{bmatrix} 7 \\ 2 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & -2 \\ 1 & 1 \end{bmatrix} \right] \quad (4.3)$$

```
> v1:=Normalize(Column(V,1),Euclidean);
```

```
v2:=Normalize(Column(V,2),Euclidean);
```

$$v1 := \begin{bmatrix} \frac{1}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} \end{bmatrix} \quad (4.4)$$

$$v2 := \begin{bmatrix} -\frac{2}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} \end{bmatrix}$$

```
> P:=<v1|v2>;
```

$$P := \begin{bmatrix} \frac{1}{5}\sqrt{5} & -\frac{2}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \end{bmatrix} \quad (4.5)$$

```
> Transpose(P).Q.P;
```

$$\begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix} \quad (4.6)$$

```
> yy:=Vector([xp,yp]);
```

$$yy := \begin{bmatrix} xp \\ yp \end{bmatrix} \quad (4.7)$$

```
> e1:=expand(Transpose(yy).Transpose(P).Q.P.yy)+Transpose(bb).P.yy+2;
```

$$e1 := 7xp^2 + 2yp^2 - 2xp\sqrt{5} + 2yp\sqrt{5} + 2 \quad (4.8)$$

```
> expand(7*(xp-sqrt(5)/7)^2+2*(yp+sqrt(5)/2)^2+2-45/14);
```

$$7xp^2 + 2yp^2 - 2xp\sqrt{5} + 2yp\sqrt{5} + 2 \quad (4.9)$$

3

```
> restart;
```

```
f:=unapply(a*x^2-b*x-a+b,x);
```

$$f := x \rightarrow ax^2 - bx - a + b \quad (5.1)$$

```
> f(-2)=6;
```

$$3a + 3b = 6 \quad (5.2)$$

```
> b0:=solve(f(-2)=6,b);
```

$$b0 := 2 - a \quad (5.3)$$

```
> x0:=solve(subs(b=b0,diff(f(x),x)),x);
```

$$x0 := -\frac{1}{2} \frac{-2+a}{a} \quad (5.4)$$

```
> factor(subs(b=b0,f(x0)));
```

$$-\frac{1}{4} \frac{(3a-2)^2}{a} \quad (5.5)$$

4

```
> restart;
```

```
f1:=4*x^2-8*x+5;
```

```
f2:=-2*(x+a)^2+b;
```

$$\begin{aligned} f1 &:= 4x^2 - 8x + 5 \\ f2 &:= -2(x+a)^2 + b \end{aligned} \quad (6.1)$$

```
> x0:=solve(diff(f1,x),x);
```

$$x0 := 1 \quad (6.2)$$

```
> y0:=subs(x=x0,f1);
```

$$y0 := 1 \quad (6.3)$$

```
> x1:=solve(diff(f2,x),x);
```

$$x1 := -a \quad (6.4)$$

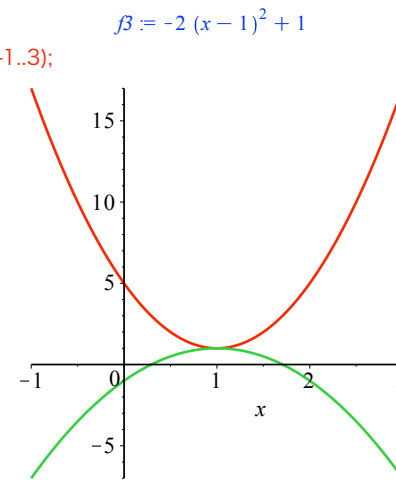
```
> y1:=subs(x=x1,f2);
```

$$y1 := b \quad (6.5)$$

```
> f3:=subs([-x1=-x0,y1=y0],f2);
```

(6.6)

```
> plot([f1,f3],x=-1..3);
```



(6.6)