1.微積分

Einstein結晶のエネルギー

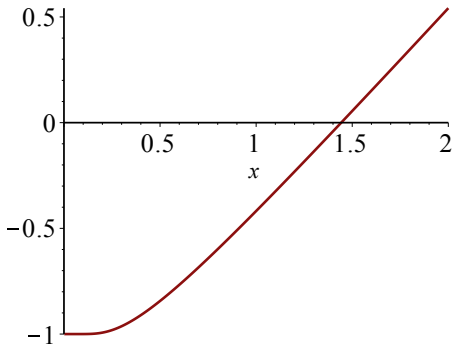
```
> restart;
  func:=x->exp(1/x)/(1-exp(-1/x));
```

$$func := x \to \frac{e^{\frac{1}{x}}}{1 - e^{-\frac{1}{x}}} \qquad (1.1.1)$$

```
> eq1:=simplify(x^2*diff(log(func(x)),x));
```

$$eq1 := -\frac{e^{\frac{1}{x}} - 2}{e^{\frac{1}{x}} - 1} \qquad (1.1.2)$$

```
> plot([eq1],x=0..2);
```



2重積分(RefDoubleInt.pdf p2, Terada,p.89)

```
> restart;
  assume(x>0);
  I1:=int(sqrt(2*x^2-y^2),y=0..x);
```

$$I1 := \frac{1}{2} x^2 + \frac{1}{4} x^2 \pi \qquad (1.2.1)$$

```
> int(I1,x=0..1);
```

$$\frac{1}{6} + \frac{1}{12} \pi \qquad (1.2.2)$$

2.線形代数

行列の対角化

```
> restart; with(LinearAlgebra):
  A:=Matrix([[1,1,3],[-1,0,1],[1,2,1]]);
> l,P:=Eigenvectors(A);
```

$$A := \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$l, P := \begin{bmatrix} 2 \\ \sqrt{3} \\ -\sqrt{3} \end{bmatrix}, \begin{bmatrix} \frac{7}{3} & \frac{2}{\sqrt{3}-1} & \frac{2}{-\sqrt{3}-1} \\ -\frac{2}{3} & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \qquad (2.1.1)$$

```
> simplify(MatrixInverse(P).A.P);
```

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & -\sqrt{3} \end{bmatrix} \qquad (2.1.2)$$

直交行列(Exam15-1Pair_References.pdf,p2, LA,p.59)

```
> restart;
  with(LinearAlgebra):
> A:=Matrix([[1/sqrt(2),a],[b,-1/sqrt(2)]]);
```

$$A := \begin{bmatrix} \frac{1}{2}\sqrt{2} & a \\ b & -\frac{1}{2}\sqrt{2} \end{bmatrix} \qquad (2.2.1)$$

```
> x1:=Column(A,1);
  x2:=Column(A,2);
```

$$x1 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ b \end{bmatrix}$$

$$x2 := \begin{bmatrix} a \\ -\frac{1}{2}\sqrt{2} \end{bmatrix} \qquad (2.2.2)$$

```
> eq1:=Transpose(x1).x1=1;
  eq2:=Transpose(x1).x2=0;
  eq3:=Transpose(x2).x2=1;
```

$$eq1 := \frac{1}{2} + b^2 = 1$$

$$eq2 := \frac{1}{2}\sqrt{2}\,a - \frac{1}{2}\,b\sqrt{2} = 0$$

$$eq3 := \frac{1}{2} + a^2 = 1 \qquad \textbf{(2.2.3)}$$

```
> s1:=solve(eq1,b);
  s2:=solve(eq3,a);
```

$$s1 := \frac{1}{2}\sqrt{2},\ -\frac{1}{2}\sqrt{2}$$

$$s2 := \frac{1}{2}\sqrt{2},\ -\frac{1}{2}\sqrt{2} \qquad \textbf{(2.2.4)}$$

```
> subs({a=s1[1],b=s1[1]},eq2);
  subs({a=s1[2],b=s1[2]},eq2);
```

$$0 = 0$$

$$0 = 0 \qquad \textbf{(2.2.5)}$$

3.数式変形

original(2014数学II,b本[2])

*(a)*ア-エ

```
> restart;
  func:=(x,p)->x^3-p*x;
```

$$func := (x, p) \to x^3 - p\,x \qquad \textbf{(3.1.1.1)}$$

```
> eq1:=diff(func(x,p),x);
```

$$eq1 := 3\,x^2 - p \qquad \textbf{(3.1.1.2)}$$

(b) 頂点 オ-ク

```
> dfunc:=unapply(eq1,(x,p));
```

$$dfunc := (x, p) \to 3\,x^2 - p \qquad \textbf{(3.1.2.1)}$$

```
> solve(dfunc(p/3,p)=0,p);
```

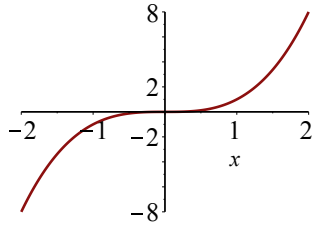
$$0,\ 3 \qquad \textbf{(3.1.2.2)}$$

```
> p0:=0;
  pp:=3;
```

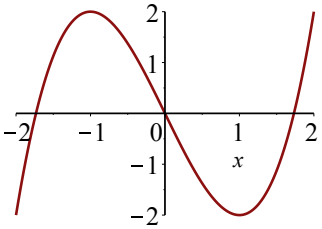
$$p0 := 0$$

$$pp := 3 \qquad \textbf{(3.1.2.3)}$$

```
> plot(func(x,p0),x=-2..2);
```



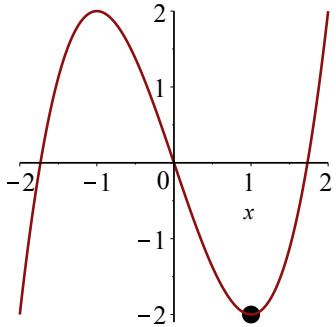

```
> plot(func(x,pp),x=-2..2);
```




```
> with(plottools):with(plots):
> [pp/3,func(1,pp)];
  pA:=disk([pp/3,func(pp/3,pp)],0.1):
  plot1:=plot(func(x,pp),x=-2..2):
```

$$[1,\ -2] \qquad \textbf{(3.1.2.4)}$$

```
> display(pA,plot1);
```



(b) 接線 ケ-ナ

```
> dfunc(b,pp)*(x-b)+func(b,pp);
```

$$\left(3\,b^2 - 3\right)(x - b) + b^3 - 3\,b \qquad \textbf{(3.1.3.1)}$$

```
> lfunc:=(x,b)->dfunc(b,pp)*(x-b)+func(b,pp)
```

$$lfunc := (x, b) \to dfunc(b, pp)\,(x - b) + func(b, pp) \qquad \textbf{(3.1.3.2)}$$

```
> lfunc(pp/3,b)=func(pp/3,pp);
```

$$(3\,b^2-3)\,(1-b)+b^3-3\,b=-2 \qquad (3.1.3.3)$$

```
> simplify(lfunc(pp/3,b)-func(pp/3,pp));
```

$$-2\,b^3+3\,b^2-1 \qquad (3.1.3.4)$$

```
> solve(lfunc(pp/3,b)-func(pp/3,pp),b);
```

$$-\frac{1}{2},\ 1,\ 1 \qquad (3.1.3.5)$$

```
> bb:=-1/2;
```

$$bb := -\frac{1}{2} \qquad (3.1.3.6)$$

```
> lfunc(x,bb);
```

$$-\frac{9}{4}\,x+\frac{1}{4} \qquad (3.1.3.7)$$

(b) 積分 ニ-ノ

```
> Dfunc:=(x,aa)->aa*(x-pp/3)^2+func(pp/3,pp);
```

$$Dfunc := (x, aa) \rightarrow aa\left(x-\frac{1}{3}\,pp\right)^2+func\left(\frac{1}{3}\,pp, pp\right) \qquad (3.1.4.1)$$

```
> solve(Dfunc(0,aa)=0,aa);
```

$$2 \qquad (3.1.4.2)$$

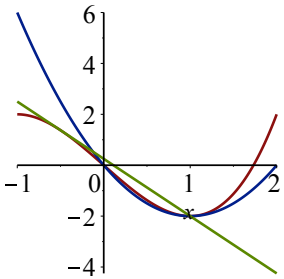
```
> aa:=2;
```

$$aa := 2 \qquad (3.1.4.3)$$

```
> expand(Dfunc(x,aa));
```

$$2\,x^2-4\,x \qquad (3.1.4.4)$$

```
> plot([func(x,pp),Dfunc(x,aa),lfunc(x,bb)],x=-1..2);
```




```
> int(lfunc(x,bb)-Dfunc(x,aa),x=0..pp/3)
```

$$\frac{11}{24} \qquad (3.1.4.5)$$

modified

(a) ア-エ

```
> restart;
  func:=(x,p)->x^3-p*x;
```

$$func := (x, p) \rightarrow x^3-p\,x \qquad (3.2.1.1)$$

```
> eq1:=diff(func(x,p),x);
```

$$eq1 := 3\,x^2-p \qquad (3.2.1.2)$$

(b) 頂点 オ-ク

```
> dfunc:=unapply(eq1,(x,p));
```

$$dfunc := (x, p) \rightarrow 3\,x^2-p \qquad (3.2.2.1)$$

```
> solve(dfunc(p/4,p)=0,p);
```

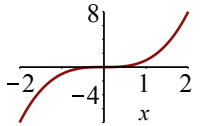
$$0,\ \frac{16}{3} \qquad (3.2.2.2)$$

```
> p0:=0;
  pp:=16/3;
```

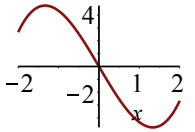
$$p0 := 0$$

$$pp := \frac{16}{3} \qquad (3.2.2.3)$$

```
> plot(func(x,p0),x=-2..2);
```



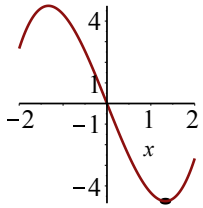

```
> plot(func(x,pp),x=-2..2);
```




```
> with(plottools):with(plots):
> [pp/4,func(1,pp/4)];
  pA:=disk([pp/4,func(pp/4,pp)],0.1):
  plot1:=plot(func(x,pp),x=-2..2):
```

$$\left[\frac{4}{3},\ -\frac{1}{3}\right] \qquad (3.2.2.4)$$

```
> display(pA,plot1);
```



(b) 接線 ケ-ナ

```
> dfunc(b,pp)*(x-b)+func(b,pp);
```

$$\left(3\,b^2-\frac{16}{3}\right)(x-b)+b^3-\frac{16}{3}\,b \qquad \textbf{(3.2.3.1)}$$

```
> lfunc:=(x,b)->dfunc(b,pp)*(x-b)+func(b,pp)
```

$$lfunc := (x,b)\to dfunc(b,pp)\,(x-b)+func(b,pp) \qquad \textbf{(3.2.3.2)}$$

```
> lfunc(pp/4,b)=func(pp/4,pp);
```

$$\left(3\,b^2-\frac{16}{3}\right)\left(\frac{4}{3}-b\right)+b^3-\frac{16}{3}\,b=-\frac{128}{27} \qquad \textbf{(3.2.3.3)}$$

```
> simplify(lfunc(pp/4,b)-func(pp/4,pp));
```

$$4\,b^2-2\,b^3-\frac{64}{27} \qquad \textbf{(3.2.3.4)}$$

```
> solve(lfunc(pp/4,b)-func(pp/4,pp),b);
```

$$-\frac{2}{3},\ \frac{4}{3},\ \frac{4}{3} \qquad \textbf{(3.2.3.5)}$$

```
> bb:=-2/3;
```

$$bb := -\frac{2}{3} \qquad \textbf{(3.2.3.6)}$$

```
> lfunc(x,bb);
```

$$-4\,x+\frac{16}{27} \qquad \textbf{(3.2.3.7)}$$

(b) 積分 ニ-ノ

```
> Dfunc:=(x,aa)->aa*(x-pp/4)^2+func(pp/4,pp);
```

$$Dfunc := (x,aa)\to aa\left(x-\frac{1}{4}\,pp\right)^2+func\left(\frac{1}{4}\,pp,pp\right) \qquad \textbf{(3.2.4.1)}$$

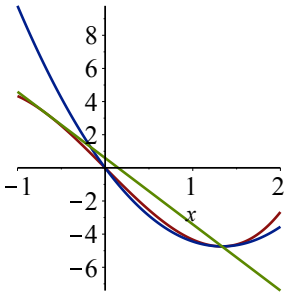
```
> solve(Dfunc(0,aa)=0,aa);
```

$$\frac{8}{3} \qquad \textbf{(3.2.4.2)}$$

```
> aa:=8/3;
```

$$aa := \frac{8}{3} \qquad \textbf{(3.2.4.3)}$$

```
> plot([func(x,pp),Dfunc(x,aa),lfunc(x,bb)],x=-1..2);
```




```
> int(lfunc(x,bb)-Dfunc(x,aa),x=0..pp/4)
```

$$\frac{352}{243} \qquad \textbf{(3.2.4.4)}$$

```
> int(lfunc(x,bb)-Dfunc(x,aa),x=0..1)
```

$$\frac{34}{27} \qquad \textbf{(3.2.4.5)}$$