## 1

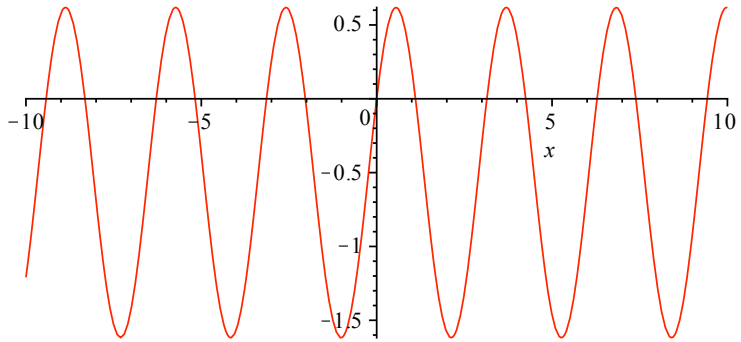
(a)

```
> normal(diff((1-sqrt(x))/(1+sqrt(x)),x));
```

$$-\frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}} \tag{1.1}$$

(b)

```
> plot(sin(2*x)-sin(x)^2,x);
```




## 2

(a)

```
> series(1/(1+x^2),x);
```

$$1 - x^2 + x^4 + O\left(x^6\right) \tag{2.1}$$

(b)

```
> int(1/(a^2*cos(x)^2+b^2*sin(x)^2),x);
```

$$\frac{\arctan\left(\frac{b\tan(x)}{a}\right)}{a\,b} \tag{2.2}$$

## 3

### (a)

```
> with(LinearAlgebra):
> A:=Matrix([[1,1,1],[a,a^2,a^3],[b,b^2,b^3]]);
```

$$A := \begin{bmatrix} 1 & 1 & 1 \\ a & a^2 & a^3 \\ b & b^2 & b^3 \end{bmatrix} \tag{3.1.1}$$

```
> factor(Determinant(A));
```

$$-a\,b\,(b-1)\,(a-1)\,(a-b) \tag{3.1.2}$$

### (b)

```
> restart;
  with(LinearAlgebra):
  B:=Matrix([[2,1,1],[1,2,1],[0,0,1]]);
```

$$B := \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2.1}$$

```
> l,v:=Eigenvectors(B);
```

$$l, v := \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3.2.2}$$

```
> MatrixInverse(v).B.v;
```

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2.3}$$

## 4

```
> P:=x->x*(x+3)*(2*x-3);
```

$$P := x \to x\,(x+3)\,(2\,x-3) \tag{4.1}$$

```
> expand(P(a+1));
```

$$2\,a^3 + 9\,a^2 + 3\,a - 4 \tag{4.2}$$

```
> eq1:=expand(P(a+1)-P(a))/2;
```

$$eq1 := 3\,a^2 + 6\,a - 2 \tag{4.3}$$

```
> solve(eq1,a);
```

$$-1 + \frac{1}{3}\sqrt{15},\ -1 - \frac{1}{3}\sqrt{15} \tag{4.4}$$

## 5

```
> num:=0;
  for i from 1 to 100 do
    if ithprime(i)>200 then break; end if;
    if ithprime(i+1)-ithprime(i)=4 then
      num:=num+1;
      print(num,ithprime(i+1),ithprime(i));
    end if;
  end do;
```

$$num := 0$$

$$1,\ 11,\ 7$$

```
                                2, 17, 13
                                3, 23, 19
                                4, 41, 37
                                5, 47, 43
                                6, 71, 67
                                7, 83, 79
                                8, 101, 97
                                9, 107, 103
                                10, 113, 109
                                11, 131, 127
                                12, 167, 163
                                13, 197, 193                    (5.1)
> num:=0;
  for i from 1 to 100 do
    if ithprime(i)>200 then break; end if;
    if (ithprime(i+1)-ithprime(i)=4) then
      num:=num+1;
      print(num,ithprime(i+1),ithprime(i));
    end if;
    if (ithprime(i+2)-ithprime(i)=4) then
      num:=num+1;
      print(num,ithprime(i+2),ithprime(i));
    end if;
  end do;
                                num := 0
                                1, 7, 3
                                2, 11, 7
                                3, 17, 13                       (5.2)
```

## 別解

```
> restart;
  prime_list:=[];
  for i from 1 to 100 do
    if ithprime(i)>200 then break; end if;
    prime_list:=[op(prime_list),ithprime(i)];
  end do:
                                prime_list := [ ]               (5.1.1)
> nmax:=nops(prime_list);
                                nmax := 46                      (5.1.2)
> num:=1;
  for i from 1 to nmax do
    for j from i+1 to nmax do
      if prime_list[j]-prime_list[i]=4 then
        print(num,prime_list[j],prime_list[i]);
        num:=num+1;
      end if;
    end do;
  end do;
                                num := 1
                                1, 7, 3
                                2, 11, 7
                                3, 17, 13
                                4, 23, 19
                                5, 41, 37
                                6, 47, 43
                                7, 71, 67
                                8, 83, 79
                                9, 101, 97
                                10, 107, 103
                                11, 113, 109
                                12, 131, 127
                                13, 167, 163
                                14, 197, 193                    (5.1.3)
```