Qualitative Shape Representation and Reasoning Based on Concavity and Tangent Point

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Abstract

We propose a method for qualitative analysis of changes in the shape of an object, focusing on the generation of concavities, tangent points, and division. We aim at qualitative treatment on changes in the shape of a cell sheet arising during organogenesis. We develop a descriptive language that qualitatively represents the shape of an object at a low level, and then a method extracting features of the object from the expression in this language. To afford a higher-level representation, we classify shapes using only the concavity and tangent point status, and the number of components. We develop transition rules governing these qualitative shape representations and show the state transitions. This enables both qualitative simulation and backward reasoning when an unexpected state arises.

1 Introduction

Recently, life science has become an important research field. It is very interesting to analyze or formalize processes in the developmental biology from a viewpoint of computer science. At the same time, such investigations will contribute the advancement of life science.

Organogenesis commences with a sphere termed an alveolus, the surface of which is covered with a sheet of cells. This sheet changes in shape via simple transformations such as folding or splitting, gradually becoming an organ. Thus, eyes, ears, and neural tubes are formed via diverse shaping of cell sheets. Although these developmental changes are continuous, it is reasonable to create a qualitative model to analyze the principal changes and their causes. In a qualitative simulation, we consider only the states at which major events occur and the transitions between them [de Kleer, 1993; Kuipers, 1993; Forbus, 2010]. However, qualitative simulation has not yet been applied to deal with changes in shape such as folding or splitting.

Let us consider an example. Figure 1 shows the organogenesis of an eye based on [Wolpert and Tickle, 2011]. The part highlighted in bold, termed the crystalline lens plate, will transform into the eye. In the states of (A) and (B), the plate is not bent, but then becomes bent, generating a concavity, when developing into state (C), and a new object termed a lens cell separates from the plate in state (D) after the entrance of the concavity is closed.

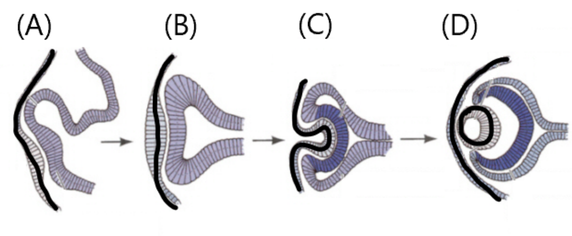


Figure 1: Organogenesis of an eye.

This can be modeled as a qualitative shape change of the cell sheet, as shown in Figure 2.

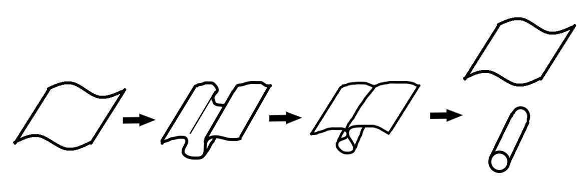


Figure 2: A model for organogenesis of an eye.

In this paper, we take an approach of Qualitative Spatial Reasoning (QSR) toward the representation and reasoning on changes in shape during organogenesis. QSR is a method used for representation and analysis without using precise numerical data. It focuses on certain aspects of an object, and reasons by reference to those aspects. It has an advantage mainly on the following points: it does not require extensive computational resources and it fits human cognition.

Over the past 30 years, many works on QSR have been published [Stock, 1997; Cohn and Hazarika, 2001; Cohn and Renz, 2007; Ligozat, 2011; Chen et al., 2013]. of which several focused on the shape representation of objects projected onto a two-dimensional plane [Leyton, 1988; Cohn, 1995; Schlieder, 1996; Galton and Meathrel, 1999; Kulik and Egenhofer, 2003; Gottfried, 2003; Gottfried, 2004; Museros and Escrig, 2004]. These objects generally featured closed boundaries and had neither an end point nor a tangent point. On the other hand, a cell sheet may connect to itself, creating a tangent point, and the sheet may be cut at that point. Such a change in shape cannot be represented using existing methods.

In this paper, we first develop a language representing the shape of an object. Our initial low-level representation employs a directed line and a point as primitive terms, providing a qualitative picture of the outline of an object. Then, we extract the characteristics of the shape from this representation. These characteristics might include the existence of concavities and/or tangent points. We thus generate a high-level abstract representation; we use predicates to this end. Next, we develop state transition rules applicable at the higher level. Using these rules, we show that a qualitative change frequently found during organogenesis can be modeled.

Moreover, if we impose certain transition rules on the symbolic representation, new characteristics may be extracted and a new, high-level state transition rule may be generated. As a result, we may find an unexpected state and determine the reason why it appeared.

This paper is organized as follows. After describing some basic concepts of graph theory and elementary geometry in Section 2, we describe shapes and the changes to be modeled in Section 3. In Section 4, we develop a language representing the shape of an object at a low level. Then, we extract the features of the shape from this expression. In Section 5, we construct a state transition system employing these features. In Section 6, we compare our work with related works and in Section 7, we show conclusions and mention our future works.

2 Preliminaries

We here summarize several basic concepts of graph theory [Harary, 1969] and elementary geometry.

A graph is defined as a pair of a set of vertices and a set of edges. For a graph, a sequence of edges $E_1, \ldots, E_n$ with E_i connecting vertices V_{i-1} and V_i ($1 \le i \le n$) is said to be *a walk*, and if $V_0 = V_n$, the walk is said to be *closed.* For a walk, if $E_i \neq E_j$ for each pair of (i, j) $(1 \le i < j \le n)$, the walk is said to be *a trail*; if $V_i \neq V_j$ for each pair of (i, j) $(0 \le i < j \le n)$, the walk is said to be *a path*. A closed trail is said to be *a circuit*. For a closed trail, if $V_i \neq V_j$ for each pair of (i, j) $(0 \le i < j \le n-1)$, then it is said to be *a cycle*. A walk that visits every edge exactly once is said to be *an Eulerian trail*. For a vertex, the number of outgoing or incoming edges is said to be *the degree* of the vertex. If zero or two vertices in a graph are of odd degree, then the graph has an Eulerian trail. An Eulerian trail that starts and ends on the same vertex is said to be *an Eulerian circuit*. A graph containing an Eulerian circuit is said to be *an Eulerian graph* (Figure 3(a)). A graph containing an Eulerian trail but not an Eulerian circuit is said to be *a semi-Eulerian graph* (Figure 3(b)).

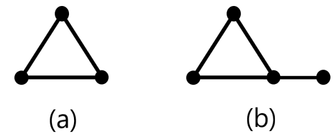


Figure 3: (a) An Eulerian graph and (b) a semi-Eulerian graph.

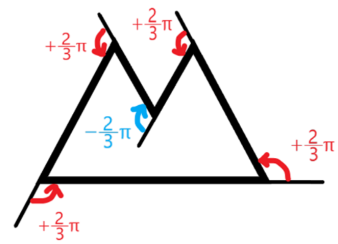


Figure 4: Exterior angle.

The outline of a polygon can be considered to be a graph. Here, we determine the direction of each edge by starting from an arbitrary vertex and tracing the boundary anticlockwise, viewing from the left. Let E_1 be an edge and E_2 be the adjacent edge of a polygon. Then, the *exterior angle* of a vertex is defined as the angle made by the extension of E_1 and the edge E_2. When the vertex is convex, the exterior angle is positive, while when the vertex is concave, the angle is negative. The sum of the exterior angles of a polygon is 2π (Figure 4).

We recall the following propositions from elementary geometry.

Proposition 1 *Let $E_1, \ldots, E_n$ be a sequence of edges, and θ_i $(1 \le i \le n)$ be the exterior angle between E_{i-1} and E_i. Then, an Eulerian graph can be drawn iff $\Sigma_{1 \le i \le n} \theta_i = 2\pi$.*

Proposition 2 *An Eulerian trail can be drawn without an intersection.*

It follows immediately that an Eulerian graph and a semi-Eulerian graph can be drawn using one stroke.

3 Shapes and changes to be modeled

We consider the shape of the cross-section of a cell sheet projected onto the two-dimensional plane.

Generally, organogenesis features three important transformations of a cell sheet: concavity generation, tangent point generation, and division. Therefore, we classify shapes depending on these features. Shapes that exhibit the same characteristics as the concavity and the tangent point are regarded as equivalent. Objects with and without end points can be handled. The orientation of a figure, its size, and the number of concavities, are ignored. We show examples of shapes which may appear by transforming a cell sheet in Figure 5.

In this figure, the shapes within the dotted rectangles are regarded as the same, whereas shapes in different rectangles are considered different.

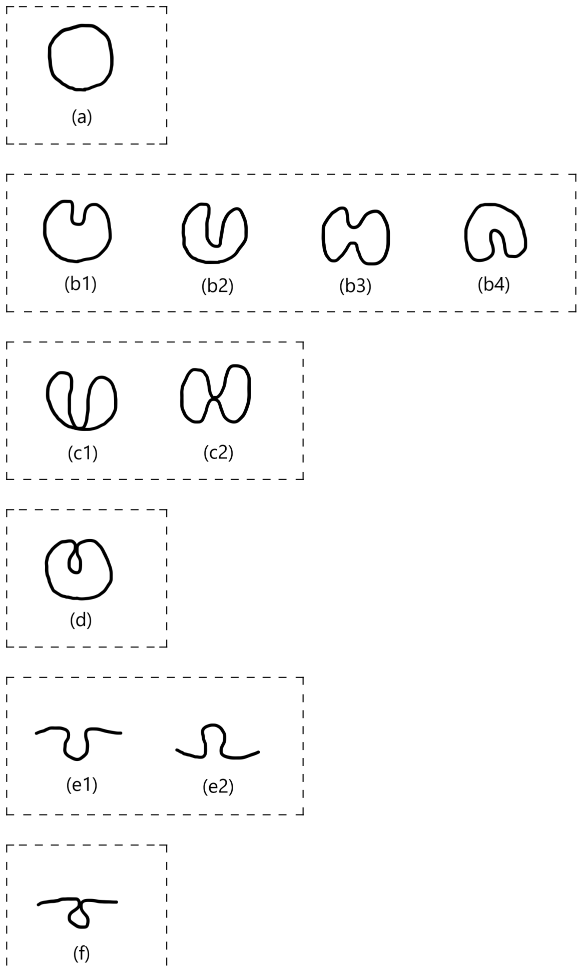


Figure 5: Examples of shapes appearing in the organogenesis process.

In fact, the ends of a cell sheet are connected to an organ but, as we investigate local changes in shape, we assume that the sheet is of finite size. A cell sheet never crosses itself, because it is a "sheet." It means that the figure can be drawn using one stroke without an intersection.

Figure 6 shows one of changes that frequently appears in the organogenesis process, to which we are going to give a qualitative representation.

4 Description Language

4.1 Language

We develop a language $\mathcal{L}$ representing the shape of the outline of an object.

$$exp ::= (seg+) \mid [seg+]$$
$$seg ::= dline \mid point$$

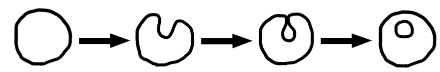


Figure 6: An example of a shape change appearing in the organogenesis process.

$$dline ::= r \mid r^+ \mid r^- \mid l \mid l^+ \mid l^-$$
$point ::= a \mid b \mid \ldots$ where $a, b, \ldots$ are constants.

To eliminate redundancy, we impose the constraint that a segment never appears immediately after the same segment.

The language $\mathcal{L}$ is a set of expressions exp, satisfying this constraint. exp is a sequence of segments surrounded by parentheses or brackets; if an exp is surrounded by parentheses, then exp is called *an open expression*; and if it is surrounded by brackets, then exp is called *a closed expression*. The sequence between parentheses or brackets is called *a sequence of the expression*. Segment seg is either $dline$ or $point$. There are six types of $dline$, and $point$ is a constant.

S, D, and P, denote the sets of all segments, all directed lines, and all points, respectively. Then, $S = D \cup P$ and $D \cap P = \emptyset$ hold.

The structure of a closed expression $[x_1 \ldots x_n]$ is cyclic, which means that $x_{i+n} = x_i$ holds for any i ($0 < i \leq n$). For an open expression $(x_1 \ldots x_n)$, x_{i+n} ($0 < i \leq n$) is undefined, denoted as $\perp$.

We introduce a function $succ$. For a sequence $x_1 \ldots x_n$, $x_{succ(i)}$ indicates the $dline$ that appears after x_i.

$$succ(i) = \begin{cases} i+1 & (\text{if } x_{i+1} \in D) \\ i+2 & (\text{if } x_{i+1} \in P) \\ \perp & (\text{if } x_{i+1} \text{ is } \perp) \end{cases}$$

For an expression e and a point p, $occur(e, p)$ indicates the number of times p occurs in e.

4.2 Semantics

An expression corresponds to a figure drawn on a two-dimensional plane. Intuitively, an expression is the trace of an outline of an object. A closed expression corresponds to a case in which the start point and the end point coincide, whereas an open expression corresponds to a case in which the points are different. $dline$ means a directed line that has $\pi/3$ steps, the lengths of which are ignored (Figure 7), and $point$ indicates an intersection.

We define the angle of rotation between two segments as a function rot from $S \times S$ to $\{n\pi/3 \mid n \in \{-2, -1, 0, 1, 2, 3\}\}$[1].

- For $d \in D$,
  $rot(r, r) = 0$. $\quad rot(r, r^+) = \pi/3$.
  $rot(r, l^+) = 2\pi/3$. $\quad rot(r, l) = \pi$.
  $rot(r, l^-) = -2\pi/3$. $\quad rot(r, r^-) = -\pi/3$.
  For the other $d, d' \in D$, $rot(d, d')$ is similarly defined.
- For $p \in P$ and $x \in S$,

[1]Here, we use a global coordinate axis to describe the outline, but a description using a relative axis is also available.

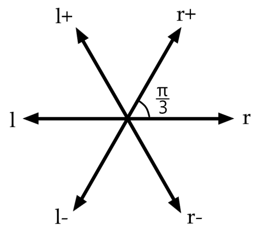


Figure 7: Directed lines.

– $rot(p, x) = 0. \quad rot(x, p) = 0.$

For a sequence $s = x_1 \ldots x_n$, if $\Sigma_{i=1}^{n} rot(x_i, x_{succ(i)}) = \pm 2\pi$, then s is said to be a *closed sequence*.

A function rev from D to D is defined as follows:

- For $d \in D, \quad rev(d) = d'$ iff $rot(d, d') = \pi$.

Intuitively, $rev(d)$ is a directed line in the direction opposite to d. Clearly, $rev(rev(d)) = d$ holds for any $d \in D$.

The language $\mathcal{L}$ can represent all shapes formed by the transformation of a cell sheet. For example, let $exp_1 = (r\ a\ l^-\ r\ l^+\ a\ r)$ and $exp_2 = [r\ l^+\ l^-\ l^+\ l^-]$ be expressions. Then, their corresponding figures are shown in Figure 8 and Figure 9, respectively.

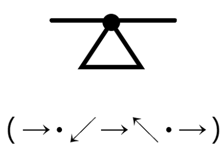


Figure 8: A figure corresponding to exp_1.

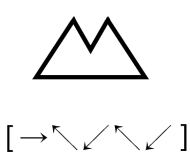
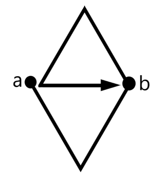


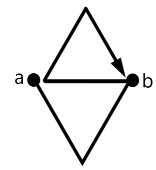
Figure 9: A figure corresponding to exp_2.

It is interesting to note that different expressions correspond to the same figure. Figure 10 is a figure equivalent to both expressions: $exp_3 = (a\ r^-\ r^+\ b\ l^+\ l^-\ a\ r\ b)$ and $exp_4 = (a\ r^-\ r^+\ b\ l\ a\ r^+\ r^-\ b)$.

Similarly, Figure 11 corresponds to both expressions: $exp_5 = (r^-\ a\ l\ r^-\ r^+\ l\ a\ r^-\ l\ r^+\ a\ r^+)$ and $exp_6 = (r^-\ a\ l\ r^-\ r^+\ l\ a\ l^-\ r\ l^+\ a\ r^+)$. However, if we draw this figure according to exp_6, we have to cross lines, which cannot be achieved by transformation of a sheet. Therefore, we will later impose conditions on an expression to eliminate such a case.

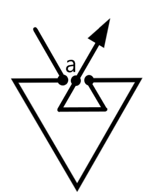


(a) Drawn by reference to exp_3.

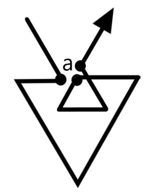


(b) Drawn by reference to exp_4.

Figure 10: A figure corresponding to exp_3 and exp_4.



(a) Drawn by reference to exp_5.



(b) Drawn by reference to exp_6.

Figure 11: A figure corresponding to exp_5 and exp_6.

4.3 Consistent expression

We discuss the conditions an expression should satisfy so that a corresponding figure that can be obtained by transformation of a sheet exists.

First, we introduce the set $Split$. For an expression e and a point p, $Split(e, p)$ is defined to be a set of closed sequences, each of which is surrounded by the same point, respectively.

$Split(e, p) =$
$\{x_i \ldots x_j \mid 1 < i < j \wedge j - i < n \wedge x_{i-1} = x_{j+1} = p\}$

For example, $Split(exp_5, a) = \{l\ r^-\ r^+\ l,\ r^-\ l\ r^+,\ l\ r^-\ r^+\ l\ a\ r^-\ l\ r^+\}$, and $Split(exp_6, a) = \{l\ r^-\ r^+\ l,\ l^-\ r\ l^+,\ l\ r^-\ r^+\ l\ a\ l^-\ r\ l^+\}$.

The following conditions ensure that segments are never superposed on the plane.

Definition 1 *Let e be an expression of which sequence is $x_1 \ldots x_n$. If e satisfies all of the following conditions, then it is called* a consistent expression.

1. $\forall i(\forall p, p' \in P(x_i = p \Rightarrow x_{i+1} \neq p'))$.
2. $\forall i(\forall d \in D(x_i = d \Rightarrow x_{i+1} \neq rev(x_i)))$.
3. *For $p \in P$, let $I(e, p) = \{i - 1 \mid 0 < i \leq n \wedge x_i = p\}$ and $O(e, p) = \{i + 1 \mid 0 \leq i < n \wedge x_i = p\}$.*
   - $\forall p \in P(\forall in \in I(e, p), out \in O(e, p), x_{in} \neq x_{rev(out)})$.
   - $\forall p \in P(\forall in, in' \in I(e, p), \forall p \in P(\forall in, in' \in I(e, p), in \neq in' \Rightarrow x_{in} \neq x_{in'})$.
   - $\forall p \in P(\forall out, out' \in O(e, p), \forall p \in P(\forall out, out' \in O(e, p), out \neq out' \Rightarrow x_{out} \neq x_{out'})$.
4. $\forall p \in P(occur(e, p) \neq 1)$.
5. *If $e = (x_1 \ldots x_n)$, then $\forall p \in P(x_1 \neq p \vee x_n \neq p)$.*
6. *If $e = [x_1 \ldots x_n]$, then $x_1 \ldots x_n$ is a closed sequence.*

7. $\forall p \in P(\forall e' = x_1 \ldots x_n \in Split(e, p),$
   $x_1 \ldots x_n$ *is a closed sequence).*

Condition 3 means that directed lines to/from a point are never crossed.

Condition 7 is imposed so that the corresponding figure can be drawn without crossing lines. For example, as each element of $Split(exp_5, a)$ is a closed sequence, exp_5 satisfies condition 7; on the other hand, as the first two elements of $Split(exp_6, a)$ are closed sequences, whereas the third is not, exp_6 does not satisfy condition 7.

For an expression in $\mathcal{L}$, we can easily obtain the corresponding graphical expression by regarding each directed line as an edge and the start and end points as vertices, respectively. Thus, from Proposition 1 the following proposition holds:

Proposition 3 *For a consistent expression e, there exists an Eulerian trail.*

It follows from Proposition 2 that a consistent expression e can be drawn without an intersection.

5 Higher-Level Analysis

5.1 Extraction of a feature

We generate an abstract representation from an expression in $\mathcal{L}$. In this higher-level analysis, we represent only the characteristics of shapes based on concavities and tangent points, together with the number of included cycles, whereas $\mathcal{L}$ represents a configuration of segments. A figure should be evaluated with respect to these three features. We ignore the relative extents of concavities and curvatures, and classify shapes using only the existence or not of a concavity and/or a tangent point, and the number of cycles.

Let e be a consistent expression. $D(e), T(e)$ and $N(e, k)$ indicate that e has a concavity, e has a tangent point to itself, and e has k cycles, respectively. We can omit e if the expression is trivial.

- $D(e)$ holds if $\exists i; (rot(x_i, x_{i+1}) \times rot(x_{i+1}, x_{i+2}) < 0)$
- $T(e)$ holds if $\exists p \in P; occur(e, p) \neq 0$
- $N(e, k)$ holds if the number of cycles of e is k when we regard the expression as a graph. Formally, k is determined as follows:

$$k = \begin{cases} \Sigma_{p \in P} max(occur(e,p) - 1, 0) \\ \qquad\qquad \text{(if } e = (x_1 \ldots x_n)) \\ \Sigma_{p \in P} max(occur(e,p) - 1, 0) + 1 \\ \qquad\qquad \text{(if } e = [x_1 \ldots x_n]) \end{cases}$$

For a consistent expression e, the characteristics can be represented using these predicates. There are four possible combinations of the truth values of D and T, but $D \wedge T$ can be omitted from consideration, as the entrance to a concave part is closed and a concavity disappears when a tangent point appears.

5.2 State transition

Next, we construct state transition rules using the predicates D, T and N.

The following properties hold: when a tangent point is generated, the number of cycles increases; and, when division occurs, the number of cycles are divided between the two resulting objects.

Thus, we obtain the following three state transition rules. The symbol '$\rightarrow$' indicates a direct transition that is *a conceptual neighbor* in the case of a single direction [Freksa, 1992].

For a consistent expression e:

- (R1) Generation of a concavity
  $\neg D(e) \wedge \neg T(e) \wedge N(e,k) \rightarrow D(e) \wedge \neg T(e) \wedge N(e,k)$
- (R2) Generation of a tangent point
  $D(e) \wedge \neg T(e) \wedge N(e,k) \rightarrow \neg D(e) \wedge T(e) \wedge N(e,k+1)$
- (R3) Division
  $\neg D(e) \wedge T(e) \wedge N(e,k) \rightarrow (\neg D(e_1) \wedge \neg T(e_1) \wedge N(e_1,k_1)) \wedge (\neg D(e_2) \wedge \neg T(e_2) \wedge N(e_2,k_2))$
  where $k = k_1 + k_2, k_1, k_2 \geq 0$

It is sufficient to consider the case for $n = 0, 1, 2$ in terms of the state transition rules. There are three constraints on state transitions: (i) when a concavity is generated, a tangent point is not generated at the same time, (ii) a tangent point is never generated without generation of a concavity, and (iii) an object without a tangent point never divides. Therefore, we have six classes of possible shapes, as shown in Table 1.

Table 1: Possible shapes.

feature \ N(k)	k=0	k=1	k=2
¬D ∧ ¬T			
D ∧ ¬T			
¬D ∧ T			

The class $\neg D \wedge T \wedge N(2)$ has two shapes. We use one of them (for example, the former) as a representative of the class. The expressions in $\mathcal{L}$ for representatives of each class are shown in Figure 12. It is easy to check that individual features can be extracted from these expressions, respectively.

Applying the above transformation rules, we obtain the state transition graphs shown in Figure 13. Note that when an object divides, we represent only the shape of each object formed, ignoring their relative positions. Such changes may be noted during most organogeneses, such as those of the lens of the eye, the semicircular canal that is transformed to the inner ear tube, and the neural tube. Thus, this state transition graph affords a qualitative model of organogenesis.

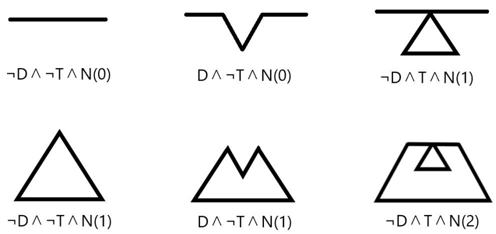


Figure 12: The expressions in $\mathcal{L}$ for each state.

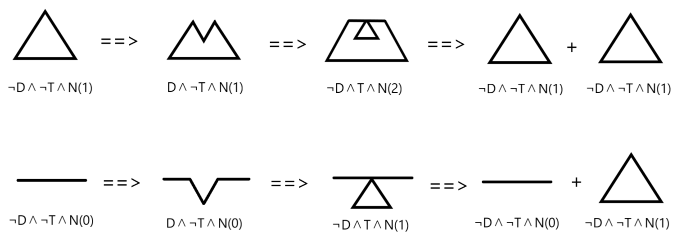


Figure 13: A qualitative model for an organogenesis process.

5.3 Granularity refinement

The class $\neg D \wedge T \wedge N(2)$ has two shapes that differ both topologically and cognitively. It is natural that these should be discriminated. Both shapes can be outcomes of a change from the class $D \wedge \neg T \wedge N(1)$. One is obtained by closing the entrance of the concavity to an external tangent, whereas the other is obtained by deepening the concavity to internalize the tangent (Figure 14).

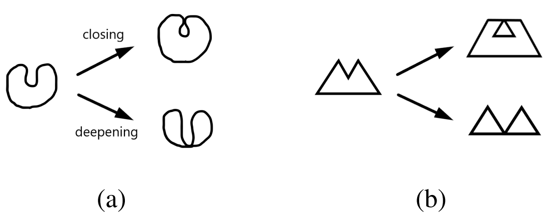


Figure 14: (a) Two different shapes of $\neg D \wedge T \wedge N(2)$ and (b) their expressions in $\mathcal{L}$.

These two figures are associated with different expressions in $\mathcal{L}$, whereas both are in the same class $\neg D \wedge T$ at the higher level. This means that the abstraction is too coarse. Therefore, we should assign intermediate granularity to the representation by introducing predicates such as external and internal tangents.

Moreover, if we consider the change of shape indicated by the expression $\mathcal{L}$, new characteristics may be extracted and a new state transition rule generated. For example, we can consider a new state transition rule for the shape change shown in Figure 15. This corresponds to the change from a closed expression $[x_1 \ldots x_n]$ to an open expression $(x_1 \ldots x_n)$ in $\mathcal{L}$. We expect that neural tube obstructions arise in this way, and that other abnormal states may develop in a similar manner.

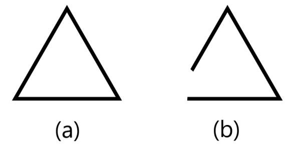


Figure 15: Possible state transition rule: (R4) Cutting.

We can describe this transition rule as follows:

- (R4) Cutting
  $\neg D(e) \wedge \neg T(e) \wedge N(e,k) \rightarrow \neg D(e) \wedge \neg T(e) \wedge N(e,k-1)$

This seems to be extended to the following rules:

- (R4') Cutting_2
  $D(e) \wedge \neg T(e) \wedge N(e,k) \rightarrow D(e) \wedge \neg T(e) \wedge N(e,k-1)$
- (R4") Cutting_3
  $\neg D(e) \wedge T(e) \wedge N(e,k) \rightarrow \neg D(e) \wedge T(e) \wedge N(e,k-1)$

However, (R4") cannot be accepted, as the figure yielded by the rule is not permitted to be a shape that a cell sheet assumes (Figure 16). The figure must be viewed as an inconsistent expression.

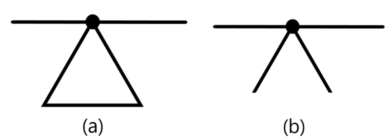


Figure 16: Unaccepted state transition rule: (R4")

.

This example indicates that we need to define a state transition in $\mathcal{L}$ or to introduce an intermediate level of abstraction.

6 Related Works

Many QSR approaches have been developed, but papers on shapes are few in number compared to those on mereological relationships or directions. The main reason is that it is difficult to identify the aspects of shapes that should be formalized.

It is natural to represent the shape of an object by tracing its boundary on a two-dimensional plane. This represents a shape as a sequence of segments, sometimes combined with the relationships between subsequent segments. Leyton developed a grammar to describe the shape of a smooth outline, based on the qualitative curvature [Leyton, 1988]. Galton and Meathrel created another shape grammar representing an outline in a similar way [Galton and Meathrel, 1999]. Unlike Leyton, the latter authors assumed that an outline consisted of a finite number of line segments.

Museros and Escrig also used line segments, but their representation additionally required a qualitative shape, an angle, or a size for each segment [Museros and Escrig, 2004]. Schlieder represented the shape of an outline via positional ordering of points on the boundary [Schlieder, 1996]. Kulik and Egenhofer developed a language to represent the characteristics of a landscape projected onto a two-dimensional plane [Kulik and Egenhofer, 2003]. They represented terrain features qualitatively using several types of primitive vectors and combinations thereof. Gottfried developed two different calculi, both of which were based on the relationships between subsequent line segments [Gottfried, 2003; Gottfried, 2004].

These languages were designed to represent only closed regions, and cannot deal with an object with an end point or a tangent point, such as that shown, for example, in Figure 8. Kulik treated a figure with end points but not a figure with a tangent point. We could extend the existing languages to represent an object with an end point or a tangent point. However, other aspects such as curvature, segment size, and concavity position that we wish to ignore are embedded in such language descriptions. Thus, we would obtain a complicated redundant representation. It seems that extensions to existing languages would not help us achieve our goal to grasp the transformation of a cell sheet, and development of a new language is a better solution.

Cohn took a different approach [Cohn, 1995], proposing a representation using relationships over regions. Convexity was considered, with a focus on the difference between the original region and its convex hull. There, subtle qualitative shape differences were represented in a hierarchical manner. The concavity and tangential point features on which we have focused can be represented by extending the his formalization. However, it would be necessary to introduce new predicates representing these features together with several axioms.

In summary, the figure used in almost all approaches had a closed boundary without a tangent point or end points. On the other hand, we allow an outline that has an end point, a tangent point, and/or a closed boundary. Another significant difference between our work and earlier papers is that we consider shape transformation including a division, while other works do not.

7 Conclusion

We have discussed a qualitative shape representation and investigated state transitions between the representations. We developed a language describing qualitative shapes in a low-level and showed a rule for extracting a higher-level representation. For a higher-level representation, we classified shapes using only the existence of a concavity, a tangent point and the number of cycles, and showed state transitions. We also discussed the granularity of expressions.

In future, we would like to define low-level state transition rules that allow us to identify hitherto unknown states. Moreover, we expect that our method can be applied not only to analysis of organogenesis but also to other applications such as exploration of change in terrain.

References

[Chen et al., 2013] Chen, J., Cohn, A., Liu, D., Wang, S., Ouyang, J., and Yu, Q. (2013). A survey of qualitative spatial representations. *The Knowledge Engineering Review*, 30(1):106–136.

[Cohn, 1995] Cohn, A. G. (1995). hierarchical representation of qualitative shape based on connection and convexity. *Spatial Information Theory. Cognitive and Computational Foundations of Geographic Information Science, International Conference COSIT'99 Stade, Germany, August 25-29, 1999 Proceedings*, pp. 311-326.

[Cohn and Hazarika, 2001] Cohn, A. G. and Hazarika, S. M. (2001). Qualitative spatial representation and reasoning: An overview. *Fundamental Informaticae*, 46(1-1):1–29.

[Cohn and Renz, 2007] Cohn, A. G. and Renz, J. (2007). *Handbook of Knowledge Representation*, chapter 13. Qualitative Spatial Reasoning, pp. 551–596. Elsevier.

[de Kleer, 1993] de Kleer, J. (1993). A view on qualitative physics, *Artificial Intelligence*, 59(1-2):105-114.

[Forbus, 2010] Forbus, K. D. (2010). *Qualitative modeling*. Wiley.

[Freksa, 1992] Freksa, C. (1992). Using orientation information for qualitative spatial reasoning. *Theories and methods of spatio-temporal reasoning in geographic space*, pp. 162-178.

[Galton and Meathrel, 1999] Galton, A. and Meathrel, R. (1999). Qualitative outline theory. *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence*, pp. 1061-1066.

[Gottfried, 2003] Gottfried, B. (2003). Tripartite line tracks qualitative curvature information. *Proceedings of the Spatial Information Theory. Foundations of Geographic Information Science, International Conference, COSIT 2003*, Ittingen, Switzerland, September 24-28. Lecture Notes in Computer Science 2825, pp. 101-117. Springer.

[Gottfried, 2004] Gottfried, B. (2004). Reasoning about intervals in two dimensions. *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, The Hague, Netherlands, October 10-13. IEEE, pp. 5324-5332.

[Harary, 1969] Harary, F. (1969). *Graph Theory*. Reading, Mass.: Addison-Wesley Pub. Co., 1969. Print.

[Kuipers, 1993] Kuipers, B. J. (1993). Qualitative simulation: then and now. *Artificial Intelligence*, 59(1-2):133-140.

[Kulik and Egenhofer, 2003] Kulik, L. and Egenhofer, M. J. (2003). Linearized terrain: languages for silhouette representations. *Spatial Information Theory. Foundations of Geographic Information Science, International Conference, COSIT 2003, Ittingen, Switzerland, September 24-28, 2003, Proceedings*, pp. 118-135.

[Leyton, 1988] Leyton, M. (1988). A process-grammar for shape. *Artificial Intelligence*, 34(2):213-247.

[Ligozat, 2011] Ligozat, G. (2011). *Qualitative Spatial and Temporal Reasoning*. Wiley.

[Museros and Escrig, 2004] Museros, L. and Escrig, M. (2004). A Qualitative Theory for Shape Representation and Matching for Design. *Proceedings of the Sixteenth Eureopean Conference on Artificial Intelligence, ECAI'2004, including Prestigious Applicants of Intelligent Systems, PAIS 2004, Valencia, Spain, August 22-27, 2004*, pp. 858-862.

[Schlieder, 1996] Schlieder, C. (1996). Qualitative Shape Representation. *Geographic Objects with Indeterminate Boundaries*, pp. 123-140.

[Stock, 1997] Stock, O. (1997). *Spatial and Temporal Reasoning*. Kluwer Academic Publishers.

[Wolpert and Tickle, 2011] Wolpert, L. and Tickle, C. (2011). *Principles of Development*. OUP Oxford.