Reasoning on Spatial Constraints over Regions

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Abstract

We discuss description and reasoning over regions. We have proposed a system called $SRCC$ that integrates integration of spatial and semantic data. $SRCC$ can describe and reason about the propagation or causality of semantic properties that hold for pairs of connected or unconnected regions. We extend $SRCC$ by introducing meta-variables so that it can handle both the propagation of a semantic property for a specific region and general propagation rules for an arbitrary region. Moreover, we revise our algorithm that checks for the unsatisfiability of a given set of formulas, so that it can derive the necessary spatial constraints if the set is satisfiable. In the revised algorithm, the relative positional relation over regions is derived a constraint from the properties that hold for a single region or interrelationship over multiple regions. For example, this can be applied to determining where a new building should be built.

Key Words: qualitative spatial reasoning, RCC, semantic data, constraint

1. Introduction

Region Connection Calculus (RCC) (Randel et al. 1992) is a calculus for qualitative spatial reasoning. In RCC, "a region" is regarded as a primitive that constructs a space, and considers relative positional relations between regions are handled such as "France and Germany touch." So far, many studies have discussed procedures for examining the consistency of a given set of positional relations (Cohn & Gotts 2002)(Renz 2002)(Li & Ying 2003), how some relations change with the passage of time (Muller 1998)(Wolter & Zakharyaschev 2000), and so on (Borgo et. al 1996)(Cohn et al. 1997). These studies have clarified interesting features of RCC.

However, RCC cannot handle semantic properties that hold in some region such as "It rains in Manhattan," or interrelations between the attributes of regions such as "Many persons living in city A work in city B." We have proposed a system called $SRCC$ that can integrate spatial and semantic data by extending RCC (Takahashi 2002)(Takahashi 2003). We have presented a sound and complete algorithm that checks the unsatisfiability of a given set of formulas in $SRCC$ and have shown $SRCC$ can provide description and reasoning for propagation over regions, such as "If it rains in city A, then many workers arrive late for work at companies in city B."

However, our previous work is insufficient in the following points: (i) the description of propagation is not general (i.e. propagations over the regions that satisfy the same condition should be described separately) and (ii) the checking algorithm for a given set of formulas does not find the constraints on the regions when the set is satisfiable. In this paper, we revise $SRCC$ with respect to these points to make it more powerful.

For the first point, $SRCC$ can treat propagation of a semantic property for a specific region, but it cannot handle a general propagation rule for an arbitrary region, since the logic of $SRCC$ is propositional. For example, assume that an event that occurred in the upper portion of a river affects the downstream portion. In this case, it is more natural to regard these two portions as an arbitrary pair of regions related to the relation governing where the river-water flows from and to, rather than as a specific pair of regions. In this paper, we introduce meta-variables to describe a general propagation rule over an arbitrary pair of regions.

For the second point, we present an algorithm that can derive the necessary spatial constraints placed on a pair of regions if the set is satisfiable that simultaneously checks for the unsatisfiability of a given set of formulas.

We can use the algorithm to determine where a new region is introduced. The algorithm finds which relation should hold between the new region and the existing ones. In addition, a relation that holds for the existing regions can also be extracted even if it is not specified.

For example, assume that you are going to move to a new house and you have to determine where to live next. This might be involve several considerations. If you have a child, you might decide to move to an area near a good school. If you have information about polluted areas, you probably want to live far from such areas. Our framework based on $SRCC$ can incorporate these considerations in the following manner. It represents the existing properties and spatial conditions using $SRCC$ formulas, it introduces a new region that corresponds to the desired features of your new residents to the existing relations, and it examines what relation should be satisfied between the new and existing areas. We demonstrate this procedure by considering where to place a filtration plant along a the polluted river.

This paper is organized as follows. First, we explain $SRCC$, and define the description language and its semantics in section 2. Then, we present the revised algorithm in section 3, and show an application in section 4. We discuss our approach in section 5, and finally we conclude in section 6.

2. SRCC

Region Connection Calculus (RCC) is one of the representatives of theories that consider a space as a set of regions, paying attention only to their relative positions, and that provide qualitative spatial representation and reasoning. It originated in Clark's theory (Clark 1981). Figure 1 shows the basic relations of RCC-5[1], a variant of RCC. These relations are jointly exhaustive and pairwise disjoint. We propose a system called $SRCC$ that extends RCC-5 so that it can handle the spatial data incorporated with semantic data.

Description Language

The description language of $SRCC$ is defined as follows.

1. region term
   (a) A region variable (denoted $X, Y, Z, \ldots$) is a term.
   (b) $f(\alpha)$ is a region term, where f is a function symbol and α is a region term[2].
2. formula
   (a) spatial formula
   $DR(\alpha, \beta)$, $PO(\alpha, \beta)$, $EQ(\alpha, \beta)$, $PP(\alpha, \beta)$ and $PPi(\alpha, \beta)$ are spatial formulas, where α and β are region terms. The formulas constructed from these using Boolean operators in the usual way are spatial formulas.
   (b) property formula
   $[\Box\alpha, G]$ and $[\Diamond\alpha, G]$ are property formulas, where α is a region term and G is a literal (of propositional logic). The formulas constructed from these using Boolean operators in the usual way are property formulas.
   (c) propagation formula
      i. $[*\alpha, G] \wedge \theta \Rightarrow [*\beta, H]$ is a propagation formula, where G and H are literals (of propositional logic), θ (sometimes not appear) is a spatial formula on α and β ($*\alpha$ denotes either $\Box\alpha$ or $\Diamond\alpha$, and we use this notation hereafter).
      ii. A formula is a propagation formula if it is obtained by substituting the meta-variable $M\alpha$ for all the occurrences of a region variable in a propagation formula.

[1]RCC-5 is defined on a coarser level of cognitive granularity than RCC-8, another variant of RCC. RCC-5 does not distinguish the relation in which regions are connected at a point and the relation in which regions are disconnected while RCC-8 makes this distinction.

[2]For simplicity, we use unary function symbols here, but we can extend the language by allowing n-ary function symbols.

We use the spatial formula $P(\alpha, \beta)$ to denote the part-of relation defined by $PP(\alpha, \beta) \vee EQ(\alpha, \beta)$, and $Pi(\alpha, \beta)$ to denote $PP(\beta, \alpha)$.

$[\Box\alpha, G]$ indicates that G holds everywhere in α, whereas $[\Diamond\alpha, G]$ indicates that G holds somewhere in α.

Spatial formulas show relations over regions. Property formulas show the properties that hold in a region. Propagation formulas show the relation between the properties of some regions those of other regions. Propagation occurs only when condition θ is satisfied. If $G = H$, then it shows that a property that holds in α is propagated to β, otherwise, the property is changed. If a propagation formula contains meta-variables, it holds for any pair of regions that satisfies θ.

There are two types of propagation: functional propagation and positional propagation.

Functional propagation is available only for regions satisfying the specified semantic relation. They are explicitly described in the form: $[*\alpha, G] \wedge \theta \Rightarrow [*f(\alpha), H]$.

Conversely, positional propagation is available for all regions satisfying some spatial relation. The following propagation formulas show positional propagation that holds for any α, β and G.

Axiomatic Schema

Ax1 $[\Box\alpha, G] \Leftrightarrow \neg[\Diamond\alpha, \neg G]$

Ax2 $[\Box\alpha, G] \Rightarrow [\Diamond\alpha, G]$

Ax3 $[\Box\beta, G] \wedge P(\alpha, \beta) \Rightarrow [\Box\alpha, G]$

Ax4 $[\Box\alpha, G] \wedge P(\alpha, \beta) \Rightarrow [\Diamond\beta, G]$

Ax5.a $[\Box\alpha, G] \wedge PO(\alpha, \beta) \Rightarrow [\Diamond\beta, G]$

Ax5.b $[\Box\beta, G] \wedge PO(\beta, \alpha) \Rightarrow [\Diamond\alpha, G]$

The propagation formula obtained by replacing α, β and G appearing in an axiom by specific region terms and a specific proposition, respectively, is said to be *an instantiation of an axiom*.

The Model

We call a regular subset of a topological space $a\ region$. A region may be multipiece or may contain a hole, but it must consist of a universal dimension.

Definition. For a set of spatial formulas, if there exists a topological space that realizes all the formulas, then the set is said to be *RCC-satisfiable*.

The formula obtained by substituting all the occurrences of a region term t in φ for a meta-variable $M\alpha$ is denoted by $\varphi[M\alpha/t]$.

Definition. If a formula $\varphi[M\alpha/t]$ contains no meta-variable, then it is said to be *a ground formula* and $[M\alpha/t]$ is said to be *a ground substitution*.

Definition. We define *a structure* $\mathcal{M} = \langle \mathbf{D}, \Sigma, \Phi \rangle$ as follows.

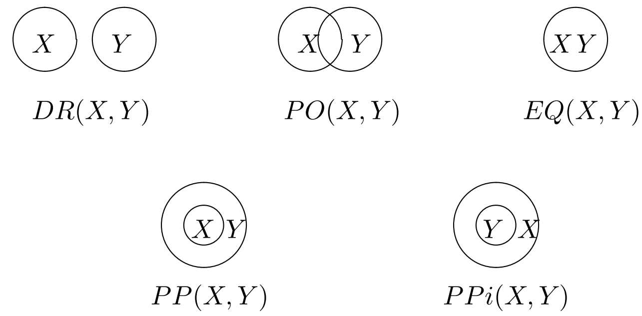

Figure 1: The basic relations of RCC-5

D is a finite set of regions, Σ is a set of spatial formulas, and Φ is the set $\{[\Box X_1, G_1], \ldots, [\Box X_n, G_n]\}$, where $X_1, \ldots, X_n$ are region variables that appear in Σ, and $G_1, \ldots, G_n$ are literals of propositional logic. For region variable X that appears in Σ, we assign an element of **D**, and for a function symbol f, we assign the map $\mathbf{D} \rightarrow \mathbf{D}$. Moreover, $\mathcal{M}$ should satisfy the following conditions (i)∼(iii).

(i) There exists a topological space $\mathcal{T}$ that realizes all the formulas in Σ.

(ii) For any X and G, $[\Box X, G] \in \Phi$ and $[\Box X, \neg G] \in \Phi$ do not hold at the same time.

(iii) If $P(\alpha, \beta)$ is realized on $\mathcal{T}$ and $[\Box \beta, G] \in \Phi$ holds, then $[\Box \alpha, G] \in \Phi$ holds.

If neither $[\Box X, G] \in \Phi$ nor $[\Box X, \neg G] \in \Phi$ holds, then this means that G holds in some part of X, and $\neg G$ holds in another part of X.

When a formula φ is true in structure $\mathcal{M}$, we denote this by $\mathcal{M} \models \varphi$. The semantics is defined as follows.

1. $\mathcal{M} \models \varphi$ iff $\varphi \in \Sigma$ where φ is a spatial formula.
2. $\mathcal{M} \models [\Box X_i, G_i]$ iff $[\Box X_i, G_i] \in \Phi$
3. $\mathcal{M} \models \neg\varphi$ iff not $\mathcal{M} \models \varphi$.
4. $\mathcal{M} \models \varphi \wedge \psi$ iff $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
5. $\mathcal{M} \models \varphi \vee \psi$ iff $\mathcal{M} \models \varphi$ or $\mathcal{M} \models \psi$
6. $\mathcal{M} \models \varphi \Rightarrow \psi$ iff not $\mathcal{M} \models \varphi$ or $\mathcal{M} \models \psi$
7. $\mathcal{M} \models \varphi \Leftrightarrow \psi$ iff both $\mathcal{M} \models \varphi$ if and only if $\mathcal{M} \models \psi$
8. $\mathcal{M} \models \varphi[M\alpha]$ iff $\mathcal{M} \models \varphi[M\alpha/X]$ for all region variable X in **D**.

Definition. Let φ be a formula. If there exists a structure $\mathcal{M}$ such that $\mathcal{M} \models \varphi$, then it is said that φ is $satisfiable$, and $\mathcal{M}$ is said to be *a model* for φ. Otherwise, φ is said to be $unsatisfiable$.

Let $\Lambda = \{\varphi_1, \ldots, \varphi_n\}$ be a set of formulas and ψ be a formula. For every model $\mathcal{M}$ in which $\varphi_1 \wedge \ldots \wedge \varphi_n$ is satisfiable, if ψ is satisfiable in $\mathcal{M}$, then ψ is said to be $a\ logical\ consequence$ of Λ, and it is denoted by $\Lambda \vdash \psi$.

Theorem. Let Λ be a set of formulas and ψ be a formula. $\Lambda \cup \{\neg\psi\}$ is unsatisfiable if and only if $\Lambda \vdash \psi$.

Proof)

($\Leftarrow$) For all $\mathcal{M}$ such that $\mathcal{M} \models \Lambda$, $\mathcal{M} \models \neg\varphi$ does not hold since $\mathcal{M} \models \Lambda \cup \{\neg\varphi\}$ does not hold. Therefore, $\mathcal{M} \models \varphi$ holds.

($\Rightarrow$) For all $\mathcal{M}$ such that $\mathcal{M} \models \Lambda$, $\mathcal{M} \models \varphi$ holds. Namely, $\mathcal{M} \models \neg\varphi$ does not hold. Therefore, $\mathcal{M} \models \Lambda \cup \{\neg\varphi\}$ does not hold.

3. Reasoning

We have devised an algorithm for checking whether a given finite set of formulas Λ is unsatisfiable (Takahashi 2003). The algorithm checks the consistency of each region, as well as the spatial consistency, that is, the existence of a topological structure that satisfies the spatial part. The consistency of each region is checked starting from the regions explicitly described in the property formulas, and then it checks other regions to which the property is propagated.

Here, we revise the algorithm so that it finds the constraint on each pair of region variables appearing in Λ when Λ is satisfiable, simultaneously it examines the unsatisfiability of Λ.

If Λ is satisfiable, the necessary condition for the satisfiability of Λ is not always given as spatial formulas. That is, the positional relations for all the pairs of region variables appearing in Λ are not specified explicitly. For example, let Λ be $\{[\Box\alpha, G], [\Box\beta, \neg G]\}$. Then, Λ is satisfiable, since there exists a topological space that satisfies $DR(\alpha, \beta)$, and both formulas in Λ can be realized in the space. However, both formulas in Λ cannot be realized simultaneously, on a topological space that satisfies $PO(\alpha, \beta)$. Therefore, $DR(\alpha, \beta)$ should hold for the satisfiability of Λ, even if the positional relation between α and β is not specified. We provide the algorithm to find these conditions.

Definition. For sets of spatial formulas $\Theta = \{\theta_1, \ldots, \theta_n\}$ and $\Theta' = \{\theta'_1, \ldots, \theta'_{n'}\}$, if $\theta_1 \wedge \ldots \wedge \theta_n \Rightarrow \theta'_1 \wedge \ldots \wedge \theta'_{n'}$, then it is said that Θ is *stronger* than Θ'.

The constraints necessary for the satisfiability of given set of formulas can be described as a set of basic relations of

RCC-5 on the pairs of region variables. In the algorithm, new constraints are added as the regions are checked, and the set of constraints becomes stronger.

The algorithm for checking unsatisfiability and deriving constraints

Λ : a given set of formulas.
Θ, Δ, Γ: the sets of spatial formulas, property formulas and propagation formulas appearing in Λ, respectively.
S_R: a set of regions apprearing in Θ.

1. $i \leftarrow 0$, $\Theta_0 \leftarrow \Theta$, $\Delta_0 \leftarrow \Delta$.
2. (RCC-satisfiability check)
   If there exists no topological space that realizes all the formulas of Θ[3],
   then Λ is unsatisfiable.
   Otherwise, continue.
3. (consistency check for each region)
   For each $\alpha \in S_R$, do the following.
   If $DR(\alpha, \alpha) \in \Theta$ or $\neg Pi(\alpha, \alpha) \in \Theta$,
   then Λ is unsatisfiable.[terminate]
   Otherwise, continue.
4. $\lambda_i \leftarrow \{\ \}$.
5. For each propagation formula $E \in \Gamma$, which is in the form $\delta \wedge \theta \Rightarrow \delta'$, do the following.
   If $\Delta_i \vdash \delta$ and $\Theta_i \vdash \theta$,
   then $\lambda_i \leftarrow \lambda_i \cup \{\delta'\}$, $\Gamma \leftarrow \Gamma - \{E\}$.
   If there exists a ground substitution σ such that $\Delta_i \vdash \delta\sigma$ and $\Theta_i \vdash \theta\sigma$,
   then $\lambda_i \leftarrow \lambda_i \cup \{\delta'\sigma\}$.
6. For each $E \in \{Ax2, \ldots, Ax5\}$, do the following.
   If there exists an instantiation of E, which is in the form $\delta \wedge \theta \Rightarrow \delta'$, such that $\Delta_i \vdash \delta$ and $\Theta_i \vdash \theta$,
   then $\lambda_i \leftarrow \lambda_i \cup \{\delta'\}$.
7. If $\lambda_i = \{\}$,
   then Λ is satisfiable.[terminate]
   Otherwise,
   $\Delta_{i+1} = \Delta_i \cup \lambda_i$.
8. (addition of constraints)
   For each $E \in \lambda_i$, do the following.
   If $E = [\Box\alpha, G]$,
   if $[\Box\beta, \neg G] \in \Delta_{i+1}$,
   then $\Theta_{i+1} \leftarrow \Theta_i \cup \{DR(\alpha, \beta)\}$.
   if $[\Diamond\beta, \neg G] \in \Delta_{i+1}$,
   then $\Theta_{i+1} \leftarrow \Theta_i \cup \{\neg Pi(\alpha, \beta)\}$.
   If $E = [\Diamond\alpha, G]$,
   if $[\Box\beta, \neg G] \in \Delta_{i+1}$,
   then $\Theta_{i+1} \leftarrow \Theta_i \cup \{\neg P(\alpha, \beta)\}$.
   Otherwise,
   $\Theta_{i+1} \leftarrow \Theta_i$.
9. $i \leftarrow i + 1$. Go to 3.

This algorithm terminates within a finite time and is complete for the unsatisfiability, if all the formulas in Λ are ground (Takahashi 2003). Termination is not guaranteed, if there exists a formula that is not ground in Λ. However, we can derive several constraints even if the algorithm does not terminate.

[3] We cannot discuss this procedure here, but it is discussed, for example, in (Renz & Nebel 1997)(Renz 2002).

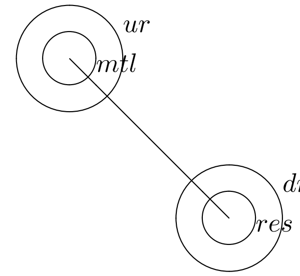

Figure 2: Inspection of the causality of contamination

Let Θ_k be the final set of constraints obtained when the algorithm for a satisfiable set Λ terminates. Then, it is clear that Θ_k is stronger than Θ. Moreover, if you add any constraint to Θ_k either in the form of $DR(\alpha, \beta)$, $\neg Pi(\alpha, \beta)$ or $\neg P(\alpha, \beta)$, the resulting set becomes inconsistent. In this sense, Θ_k is the strongest set of constraints.

4. Application

We apply the algorithm presented in the previous section to an example that involves the inspecting the causality of contamination (Figure 2).

Consider a river with a metal manufacturing factory in an upstream area, and a resident area downstream. Since the factory contaminates its environment, we plan to build the filtration plant to prevent chemicals from reaching the residents. Where should we build a filtration plant?

We describe this problem in $SRCC$. mtl, ur, dr, res and $filter$ are region variables that denote the metal manufacturing factory, the upstream area of the river, the downstream area of the river, the residents and filtration plant, respectively. The propositions $Contam$ and $Chem$ show the properties "being contaminated by the factory" and "chemicals are detected," respectively. The function $flow$ maps the region from the upstream portion of the river to the region to which the river flows. For simplicity, we assume there are no effects unless explicitly represented.

Then, the problem is formalized as follows:

$\varphi_1 : PP(mtl, ur)$
$\varphi_2 : PP(res, dr)$
$\varphi_3 : EQ(flow(ur), filter)$
$\varphi_4 : EQ(flow(filter), dr)$
$\varphi_5 : [\Box mtl, Contam]$
$\varphi_6 : [\Diamond M\alpha, Contam] \wedge \neg EQ(M\alpha, filter)$
$\quad\Rightarrow [\Box flow(M\alpha), Chem]$
$\varphi_7 : [\Diamond filter, Chem] \Rightarrow [\Box flow(filter), \neg Chem]$
$\varphi_8 : [\Box mtl, Contam] \Rightarrow [\Box mtl, Chem]$

The conclusion is represented as follows:

$\psi : [\Box res, \neg Chem]$

φ_6 and φ_7 are propagation formulas. φ_6 containing a meta-variable $M\alpha$ shows the general property of the propagation of contamination, and φ_7 shows the property with respect to a specific region $filter$. It denotes that the property of $Chem$ changes to $\neg Chem$ in the region to which the river flows after passing $filter$. Note that, no constraint on the position of $filter$ is specified explicitly.

We apply the algorithm shown in the previous section. In this case, instead of proving $\varphi_1 \wedge \ldots \wedge \varphi_8 \vdash \psi$ directly, we first prove the unsatisfiability of $\{\varphi_1, \ldots, \varphi_8\} \cup \{\neg\psi\}$. Second, to guarantee that the inference makes sense, we prove the satisfiability of $\{\varphi_1, \ldots, \varphi_8\}$. Simultaneously, we derive the spatial constraints necessary for satisfiability.

The proof for the unsatisfiability of $\{\varphi_1, \ldots, \varphi_8\} \cup \{\neg\psi\}$

$\Theta_0 = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$.
$\Delta_0 = \{[\Box mtl, Contam], [\Diamond res, Chem]\}$.

$\Theta_1 = \Theta_0$.
$\Delta_1 = \Delta_0 \cup \{[\Box mtl, Chem], [\Diamond mtl, Contam], [\Diamond dr, Chem]\}$.

$\Theta_2 = \Theta_1$.
$\Delta_2 = \Delta_1 \cup \{[\Diamond mtl, Chem], [\Diamond ur, Contam]\}$.

$\Theta_3 = \Theta_2$.
$\Delta_3 = \Delta_2 \cup \{[\Box flow(ur), Chem], [\Diamond ur, Chem]\}$.

$\Theta_4 = \Theta_3$.
$\Delta_4 = \Delta_3 \cup \{[\Box filter, Chem], [\Diamond flow(ur), Chem]\}$.

$\Theta_5 = \Theta_4$.
$\Delta_5 = \Delta_4 \cup \{[\Diamond filter, Chem]\}$.

$\Theta_6 = \Theta_5 \cup \{ DR(mtl, flow(filter)), \neg P(ur, flow(filter)), DR(flow(ur), flow(filter)), DR(filter, flow(filter)) \}$.
$\Delta_6 = \Delta_5 \cup \{[\Box flow(filter), \neg Chem]\}$.

$\Theta_7 = \Theta_6 \cup \{DR(mtl, dr), DR(flow(ur), dr), \neg P(ur, dr), D(filter, dr), \neg Pi(dr, dr)\}$.
$\Delta_7 = \Delta_6 \cup \{[\Diamond flow(filter), \neg Chem], [\Box dr, \neg Chem]\}$.

As for Θ_7, from $[\Box dr, \neg Chem] \in \lambda_7$ and $[\Diamond dr, Chem] \in \Delta_6$, $\neg Pi(dr, dr)$ is derived as a new constraint. Therefore, Λ is unsatisfiable.

The proof for the satisfiability of $\{\varphi_1, \ldots, \varphi_8\}$

$\Theta_0 = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$.
$\Delta_0 = \{[\Box mtl, Contam]\}$.

$\Theta_1 = \Theta_0$.
$\Delta_1 = \Delta_0 \cup \{[\Box mtl, Chem], [\Diamond mtl, Contam]\}$.

$\Theta_2 = \Theta_1$.
$\Delta_2 = \Delta_1 \cup \{[\Diamond mtl, Chem], [\Diamond ur, Contam]\}$.

$\Theta_3 = \Theta_2$.
$\Delta_3 = \Delta_2 \cup \{[\Box flow(ur), Chem], [\Diamond ur, Chem]\}$.

$\Theta_4 = \Theta_3$.
$\Delta_4 = \Delta_3 \cup \{[\Box filter, Chem], [\Diamond flow(ur), Chem]\}$.

$\Theta_5 = \Theta_4$.
$\Delta_5 = \Delta_4 \cup \{[\Diamond filter, Chem]\}$.

$\alpha \setminus \beta$	ur	mtl	dr	res	$filter$
ur	EQ	PPi	¬ **P**	¬ **P**	ANY
mtl	PP	EQ	**DR**	**DR**	**DR**
dr	¬ **Pi**	**DR**	EQ	PPi	**DR**
res	¬ **Pi**	**DR**	PP	EQ	**DR**
$filter$	ANY	**DR**	**DR**	**DR**	EQ

Table 1: The specified relation $R(\alpha, \beta)$ between regions

$\Theta_6 = \Theta_5 \cup \{ DR(mtl, flow(filter)), \neg P(ur, flow(filter)), DR(flow(ur), flow(filter)), DR(filter, flow(filter)) \}$.
$\Delta_6 = \Delta_5 \cup \{[\Box flow(filter), \neg Chem]\}$.

$\Theta_7 = \Theta_6 \cup \{DR(mtl, dr), DR(flow(ur), dr), \neg P(ur, dr), D(filter, dr)\}$.
$\Delta_7 = \Delta_6 \cup \{[\Diamond flow(filter), \neg Chem], [\Box dr, \neg Chem]\}$.

$\Theta_8 = \Theta_7$.
$\Delta_8 = \Delta_7 \cup \{[\Diamond dr, \neg Chem]\}$.

The algorithm terminates. Simultaneously, we obtain the constraints:

$\Theta_8 = \Theta_0 \cup \{ DR(mtl, flow(filter)), \neg P(ur, flow(filter)), DR(flow(ur), flow(filter)), DR(filter, flow(filter)), DR(flow(ur), dr), DR(filter, dr), DR(mtl, dr), \neg P(ur, dr) \}$

The resulting relations are shown in Table 1. This is the necessary condition for the satisfiability of Λ. The bold face letters show the derived constraints. For simplicity, we regard $flow(ur)$ and $flow(filter)$ as $filter$ and dr, respectively, in the table.

As a result, the following can be deduced, for example.

1. From $DR(filter, flow(filter))$, a filtration plant should be built somewhere far from the downriver area.
2. From $DR(mtl, dr)$, we found that if mtl and dr share a common part, then construction of a filtration plant is meaningless.
3. As there are no spatial constraints between ur and $filter$, a filtration plant can be built in the upper part of the river.

5. Discussion

Although many studies have examined formal methods of spatial reasoning, few have integrated spatial and semantic data.

Eschenbach proposed predication calculus (Eschenbach 1999) that can handle the semantic properties of regions in addition to their mereological and topological properties by introducing predicators that mean “somewhere in a region” and “everywhere in a region.”

Although their paper discussed the representation of semantic properties, no inference system was described. They presented a composition table for reasoning spatial relations, but did not describe any procedures to check the validity or unsatisfiability of a set of formulas. On the other hand, we present here an algorithm that checks unsatisfiability and derives the constraints.

An attempt to incorporate size with a spatial relation was presented in (Gerevini & Renz 1998), which introduced the binary operators $<, =, >$ to handle the size of regions qualitatively. The idea of utilizing relative size as a constraint to determine the spatial relations between disconnected regions is analogous to our approach, although the method that they adopted cannot be applied to $SRCC$ directly.

Viewing a property that holds everywhere or somewhere in a region is a way of indicating that a region consists of points. From this viewpoint, the specific property is satisfied in some part and it is not in another part of a single region. The idea that two counter properties may hold in the same region is similar to the egg-york model (Cohn & Gotts 1996) and scrambled egg (Guesgen 2002). Both of these models are introduced for treating vague boundaries, while in our model, we do not know whether every point in a region may or may not satisfies a specific property. In this sense, our model can be regarded as an extension of the scrambled-egg model.

6. Conclusion

In this paper, we have proposed a system that extends $SRCC$ by introducing meta-variables and shown that it can handle both the propagation of semantic properties of a specific region and general propagation rule for an arbitrary region.

We have also presented an algorithm that derives the necessary spatial constraints for satisfiability if the set is satisfiable that simultaneously checks for the unsatisfiability of a given set of formulas. The constraint is in the form of basic relations of RCC-5 on the pairs of region variables. We have applied the algorithm to an example of contamination problem and shown that it can derive spatial constraints that should be satisfied. We can use it to determine where a new region is introduced.

In future, we will explore the theoretical aspects of $SRCC$ more deeply, and discuss the limits of its expressive power. We will also consider integrating temporal reasoning into $SRCC$.

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