The set covering problem (SCP) calls for a minimum cost family of subsets from \( n \) given subsets, which together covers the entire ground set. In this paper, we propose a local search algorithm for SCP, which has the following three characteristics. (1) The use of 3-flip neighborhood, which id's the set of solutions obtainable from the current by exchanging at most three subsets. As the size of 3-flip neighborhood is \( O(n^3) \), the neighborhood search becomes expensive if implemented naively. To overcome this, we propose an efficient implementation that reduces the number of candidates in the neighborhood without sacrificing the solution quality. (2) We allow the search to visit the infeasible region, and incorporate the strategic oscillation technique realized by adaptive control of penalty weights. (3) The size reduction of the problem by using the information from the Lagrangian relaxation is incorporated, which is indispensable for solving very large instances. According to computational comparisons, on benchmark instances with other existing heuristic algorithms for SCP, our algorithm performs quite effectively for various types of problems, especially for very large-scale instances.