Hybrid Reasoning Using Weighted Bipolar Argumentation Framework for Legal Simulation

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Abstract. This paper presents a method that computes the strength of an argument using a weighted bipolar argumentation framework (WBAF) for a legal simulation system. We construct a system that suggests the most advantageous path to take when seeking to persuade an opponent by argumentation; in particular, we show the extent to which a claim shared by agents on the proponent side is strengthened or weakened throughout the argumentation. We first compute the strengths of the effect of the support relations, then combine the arguments connected by such relations into a meta-argument. Subsequently, the strengths of the effect of the attack relations are computed using the meta-WBAF. Our method has three characteristics: (i) we manage a set-support that reflects the relation between the effect of a law and the required presupposed ultimate facts; (ii) an argument does not always lose its strength when it is defended, that is, when an opponent's argument is attacked by a proponent's argument; (iii) the strengths of the effect of the attack relations are not determined by the individual attack arguments but by all arguments connected by support relations. We formalize the computation method and show its properties.

Keywords: argumentation, knowledge representation, weighted bipolar argumentation framework

1 Introduction

An abstract argumentation framework [13], which consists of a set of arguments and a relation over the set, is a powerful tool when managing inconsistencies. There are many promising applications including legal reasoning [7].

In general, a law is expressed in a form such that, if the several required presupposed ultimate facts hold, then the law is effective unless an exception occurs. We can represent this process using an (acyclic) bipolar argumentation framework (BAF) by regarding the condition part as a support and the exception as an attack on the law.

For example, consider a law regarding a compensation for damages (which is modified for the explanation). If the contract is unfulfilled in line with purpose (B1), a damage occurs (B2), and there is a causality between the damage and the opponent's behavior (B3), then the claim for the damages based on breach of contract is available (A1), unless the date of unfulfillment is beyond the prescription of three years (D1). In this example, it can be represented as a BAF in which B1, B2 and B3 together support A1, and D1 attacks A1.

Kawasaki et al. developed a hybrid reasoning method using a BAF to create a legal consultation system [16]. Two types of reasoning are available: what law is applicable when a litigator is aware of facts in a bottom-up manner, and what evidence is required to apply a specific law in a top-down manner. The system helps the user to gradually explore the range of legal knowledge by repetitive execution of these two types of reasoning in succession. During top-down reasoning, alternative reasoning paths may exist. The selection is left to the user, but it is preferable to obtain a "better" solution. In the above example, if a litigator is initially aware of B1 and B2, then she tries to prove B3 to apply A1; but if there is another law possibly to be applied, then she may choose it, and it may be hard to show a strong evidence to apply the chosen law. Thus, some criteria must be applied when choosing a "better" solution.

In this paper, we present a method by which an argument can be evaluated using a weighted bipolar argumentation framework (WBAF). WBAF is a BAF in which each argument is assigned its intrinsic weight, and the strength of arguments are calculated using the weight. We aim to construct a system that suggests the more advantageous path when seeking to persuade an opponent by argumentation.

So far, several methods of argument evaluation using a weighted argumentation framework have been proposed [2, 4–6, 8, 9, 14, 15, 17]. We extend these existing methods to treat a legal simulation.

Our method has the following three characteristics comparing with existing works. The differences lie principally in the purposes of the works: we seek to compare the strengths of arguments, whereas earlier works sought to define the rate of acceptability of arguments.

First, we manage a set-support, whereas the earlier studies did not. A set-support shows the relation between the effect of a law and a set of required presupposed ultimate facts, which is essential to represent a law.

Second, we calculate a more reasonable strength of an argument that is attacked. An argument does not always lose its strength, when it is defended, that is, when an opponent's argument is attacked by a proponent's argument. In existing approaches, the strength of an argument is always weakened, which is counter-intuitive.

Consider the following example.

I have suffered 50,000,000 yen of damage (A).
Your damage seems to be less than 10,000,000 yen (B).
Here is the document proving the 50,000,000 damage (C).

B is a weak counter-argument against A, whereas C is a strong counter-argument against B. Thus, it is reasonable to think A is strengthened by this argumentation.

Third, we calculate the value afforded by argumentation on the basis of the collective arguments for or against the principal claim, rather than by reference to the individual arguments. This is because we seek the extent by which a claim shared by agents on the proponent side is strengthened or weakened. In the existing methods, an attack on an argument connected by support relations is not considered to be an attack on a collective argument; earlier works distinguish the location of the attacked node and calculate the value of each node.

For example, consider the WBAF regarding a request for reimbursement, which is a modified version of the example shown in [18] (Figure 1).

In the figures hereinafter, a simple arrow indicates a support and an arrow with a cutting edge indicates an attack. A dashed rectangle shows a set-supporter; however, if this is a singleton, the rectangle is omitted.

Arguments A, B and C are made by the proponent side, whereas arguments D, E, F, G and H are made by the opponent side. The extents of damage in both cases are identical in terms of the claim for money, because both attacks are against the claim. It is desirable that the strength of a supporting argument is evaluated in terms of whether it is representative of the collective argument. We consider that evaluation on the entire presentations are preferable when considering the effectiveness of a particular law.

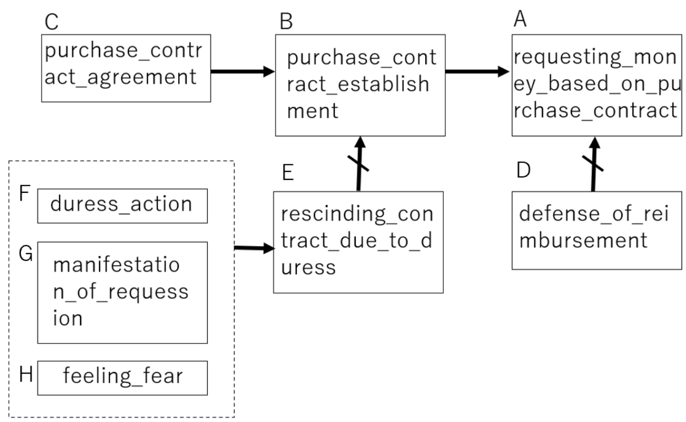

Fig. 1. The WBAF for the claim for reimbursement.

In this paper, we present a method for evaluating an argument that has these three characteristics. We separately evaluate the effects of the support and attack relations. We first combine the nodes that are connected only by support relations and compute the strength of the combination; we create a meta-WBAF lacking a support relation. We then compute the strength of the root node of the meta-WBAF. We formalize this method and describe its properties.

This paper is organized as follows. In Section 2, we describe the basic concepts of our weighted bipolar argumentation framework. In Section 3, we give the details of our evaluation method. In Section 4, we compare our method with existing ones. Finally, in Section 5, we present our conclusions and discuss our planned future work.

2 Basic Concepts

A bipolar argumentation framework (BAF) [1] is an extension of the abstract argumentation framework of Dung [13]; both attack and support relations over arguments are included. In this paper, we define a support relation from a set of arguments to a single argument that is in a form representing the law. A weighted bipolar argumentation framework (WBAF) is a BAF in which an intrinsic weight is assigned to each argument.

Definition 1 (WBAF). Weighted Bipolar Argumentation Framework *is defined as a quadruple* $\langle AR, ATT, SUP, w\rangle$ *where* AR *is a non-empty finite set of arguments,* $ATT \subseteq AR \times AR, SUP \subseteq 2^{AR} \setminus \emptyset \times AR$ *and* w *is a function from* AR *to* $\mathbb{R}$*.*

An element $(A, B) \in ATT$ is termed *an attack* and an element in A is termed *an attacker* of B. An element $(SA, B) \in SUP$ is termed *a support* and an element in SA is termed *a set-supporter* of B.

A WBAF can be represented as a weighted graph. Hereinafter, we sometimes use the terms “node” and “edge,” rather than “argument” and “relation,” respectively.

Definition 2 (support/attack path). *Let* $\langle AR, ATT, SUP, w\rangle$ *be a WBAF.*
(i) A sequence of nodes $\langle A_0, A_1, \ldots, A_k\rangle$ *where* $A_0, A_i \in AR$ *and* $\exists SA; (SA, A_{i-1}) \in SUP$ *and* $A_i \in SA$ *(*$1 \leq i \leq k$*) is said to be* a support path*.*
(ii) A sequence of nodes $\langle A_0, A_1, \ldots, A_k\rangle$ *where* $A_0, A_i \in AR$ *and* $(A_i, A_{i-1}) \in ATT$ *(*$1 \leq i \leq k$*) is said to be* an attack path*.*

For a support/attack path $p = \langle A_0, \ldots, A_k\rangle$ of the WBAF, we represent $A_i \in p$, while $|p|(= k)$ denotes the length of the path; $start(p)$ and $end(p)$ denote A_k and A_0, which are termed *the start node* and *the end node* of p, respectively.

3 Evaluation of an Argument

3.1 Outline of the procedure

In this paper, we assume that a WBAF is acyclic, since a law should be written in an acyclic manner.

For any specific argument in WBAF, we compute its strength. We consider only lower-level arguments on its lower stream, since it is affected only by its supporters and attackers.

We execute the following procedure when evaluating an argument.

1. Combine the nodes that are connected by only support relations to create a meta-argument and compute the strength of the meta-argument.
2. Create a meta-WBAF lacking a support relation.
3. Compute the strength of the root node of the meta-WBAF.

3.2 Evaluation of a supporting relation

Definition 3 (maximal support path). *Let* $\langle AR, ATT, SUP, w\rangle$ *be a WBAF. A support path* $p = \langle A_0, \ldots, A_k\rangle$ *is said to be* maximal support path to A_0 *if* $\neg\exists SA_{k+1}; (SA_{k+1}, A_k) \in SUP$*.*

Example 1. Figure 2 shows an example of a WBAF, $WBAF_1$. The figures attached to nodes indicate their weights. In $WBAF_1$, there are two maximal support paths to b: $\langle b, d, g\rangle$ and $\langle b, e, h\rangle$; and two maximal support paths to c: $\langle c, f\rangle$ and $\langle c, i\rangle$.

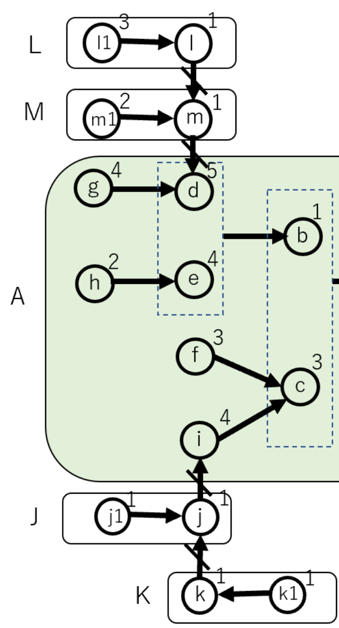

Fig. 2. *WBAF*$_1$: An example of WBAF.

Definition 4 (s-bundle). *Let* $\langle AR, ATT, SUP, w\rangle$ *be a WBAF. For* $A_0 \in AR$*, if* $\neg\exists(SA_0, B) \in SUP; A_0 \in SA_0$*, then a set of maximal support paths to* A_0 *is termed* a bundle of attackers of A_0, (s-bundle, for brevity)*, and* A_0 *is termed* a key of the s-bundle*, and* $end(p)$ *for each maximal support path* p *in the s-bundle is termed* a leaf of the s-bundle*.*

Intuitively, an s-bundle includes all nodes directly/indirectly supporting the key, as well as the key itself. They will later be combined into a single meta-argument.

Example 2. (Cont'd) In Figure 2, there are five s-bundles: L, M, A, J and K, which are indicated by capsules. The s-bundle A includes four maximal support paths: $\langle a, b, d, g\rangle$, $\langle a, b, e, h\rangle$, $\langle a, c, f\rangle$ and $\langle a, c, i\rangle$. The s-bundles L, M, J and K include one maximal support path, $\langle l, l1\rangle$, $\langle m, m1\rangle$, $\langle j, j1\rangle$, and $\langle k, k1\rangle$, respectively.

The effects of the arguments gradually decrease as they move further from the key. If an argument has several set-supporters, then the strongest set-supporter determines the strength of the supported argument. The strength of a set-supporter is the mean of the strengths of all arguments in the set, since a support becomes effective when all arguments in the set collectively hold.

Definition 5 (evaluation of an argument regarding support). *Let $\langle AR, ATT, SUP, w\rangle$ be a WBAF. The strength of an argument A is defined as follows, where r is the number of edges from the key of the s-bundle to A.*

$$str_s(A) = \begin{cases} w(A)/r & (A \text{ is a leaf of an s-bundle}) \\ max\{v_s(SA) \mid (SA, A) \in SUP\} + w(A) & (A \text{ is the key of an s-bundle}) \\ \dfrac{max\{v_s(SA) \mid (SA, A) \in SUP\} + w(A)}{r} & (otherwise) \end{cases}$$

Here, for a set-supporter SA, its value is defined as follows.

$$v_s(SA) = \sum_{a \in SA} \frac{str_s(a)}{|SA|}$$

Example 3. (Cont'd) The calculated strengths of arguments in $WBAF_1$ are as follows:

$str_s(a) = 43/8$. $str_s(b) = 15/4$. $str_s(c) = 5$ $str_s(d) = 19/6$. $str_s(e) = 7/3$.
$str_s(f) = 3/2$. $str_s(g) = 4/3$. $str_s(h) = 2/3$. $str_s(i) = 2$. $str_s(j) = 2$.
$str_s(k) = 2$. $str_s(l) = 4$. $str_s(m) = 3$. $str_s(j1) = 1$. $str_s(k1) = 1$.
$str_s(l1) = 3$. $str_s(m1) = 2$.

Definition 6 (the strength of s-bundle). *Let A_0 be the key of s-bundle S. The strength of S is defined to be $str_s(A_0)$.*

3.3 Creation of a meta-WBAF

We create a meta-WBAF lacking a support relation (an mWBAF, for brevity) by regarding each s-bundle as a meta-argument. For each s-bundle of value W, we create S as a meta-argument with weight W, and change an attack from/to a node included in the s-bundle to an attack from/to S. Then, we obtain an mWBAF $\langle AR, ATT, \emptyset, w\rangle$ such that $AR = \{S_1, \ldots, S_n\}$ where each S_i $(1 \leq i \leq n)$ is a new node corresponding to each s-bundle of which the strength is St_i. $ATT = \{(S_i, S_j)|(A, B) \in ATT, A \in p_i, p_i \in S_i, B \in p_j, p_j \in S_j\}$ and $w(S_i) = St_i$.

Example 4. (Cont'd) The mWBAF constructed from $WBAF_1$ is shown in Figure 3.

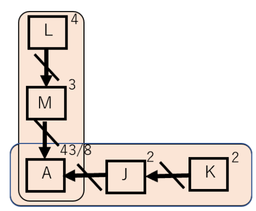

Fig. 3. The meta-WBAF of $WBAF_1$.

3.4 Evaluation of an attack relation

Next, we describe the treatment of attack relations in an mWBAF.

A maximal attack path is the longest path by which nodes are connected through attacks in a straight line without a branch.

Definition 7 (maximal attack path). *Let $\langle AR, ATT, \emptyset, w\rangle$ be an mWBAF. An attack path $p = \langle A_0, \ldots, A_k\rangle$ is said to be* a maximal attack path to A_0*, if $\neg\exists B; (B, A_k) \in ATT$ and $\neg\exists B; B \neq A_i, (B, A_{i-1}) \in ATT$ $(2 \leq i \leq k)$.*

For a maximal attack path $p = \langle A_0, \ldots, A_k\rangle$, $start(p)$ is a leaf and $end(p)$ is either the root node or a node that has a branch.

Example 5. Figure 4 shows an example of an mWBAF, $WBAF_2$. In this figure, a white node is an argument of a proponent and a blue node is an argument of an opponent. Assume that the weights of nodes are $w(f) = 2, w(g) = 4, w(h) = 3$; the weights are set at 1 for all other nodes. In $WBAF_2$, there are two maximal attack paths to e: $\langle e, f, g, h\rangle$ and $\langle e, j, k\rangle$. They are depicted by the capsules in the figure.

The value of a maximal attack path to A_0 is computed using the weights of the nodes along the path. The value is strengthened/weakened depending on the side of the agent making the argument; if the argument is made by a proponent, its weight is added; if the argument is made by an opponent, its weight is subtracted. As the proponent and the opponent advance their arguments in succession, their effects on the end node of the path are calculated in

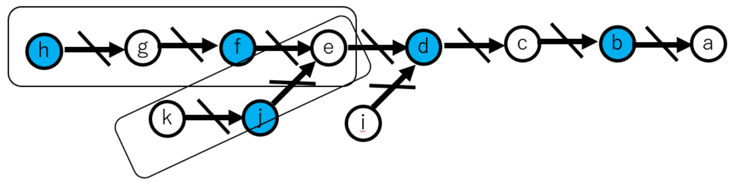

Fig. 4. $mWBAF_2$: An example of an mWBAF.

pairs. Moreover, with increasing node distance from the end node, the argument effectiveness is reduced.

Definition 8 (evaluation of maximal attack path). *For a maximal attack path $p = \langle A_0, \ldots, A_k \rangle$ of mWBAF, let r_j be the number of edges from the root node of the mWBAF to A_j $(1 \leq j \leq k)$. Then, the value of p is defined as follows.*

$$v_a(p) = \sum_{j=1}^{k} (-1)^{r_j} \cdot \frac{w(A_j)}{\lceil j/2 \rceil}$$

Example 6. (Cont'd) For an mWBAF $WBAF_2$, the values of two maximal attack paths to e are calculated as follows: $v_a(\langle e, f, g, h \rangle) = -\frac{2}{\lceil 1/2 \rceil} + \frac{4}{\lceil 2/2 \rceil} - \frac{3}{\lceil 3/2 \rceil} = 1/2$, $v_a(\langle e, j, k \rangle) = -\frac{1}{\lceil 1/2 \rceil} + \frac{1}{\lceil 2/2 \rceil} = 0$.

An a-bundle is a set of maximal attack paths that share the end node.

Definition 9 (a-bundle). *Let $\langle AR, ATT, \emptyset, w \rangle$ be an mWBAF. For $A_0 \in AR$, a set of maximal attack paths to A_0 is termed* a bundle of attackers of A_0, (a-bundle, for brevity)*, and A_0 is termed* a key of the a-bundle.

Similar to the management of support relations, we combine each a-bundle into one node. The strength of an a-bundle is defined using the values of the maximal attack paths itself and its intrinsic weight. Unlike a support relation, the definition depends on the proponent/opponent side of the key A_0 of the a-bundle. Thus, we consider the number of edges between the root node of the mWBAF and A_0. If the number is even, A_0 is given by a proponent, and the agent must defeat all counterarguments; if the number is odd, A_0 is given by an opponent, and the agent can choose the argument that is most advantageous.

Definition 10 (the strength of a-bundle). *Let A_0 be the key of a-bundle and r_0 be the number of edges between the root node of the mWBAF and A_0. Then, the strength of A_0 is defined as follows.*

$$str_a(A_0) = \begin{cases} \sum_{end(p)=A_0} (v_a(p)) + w(A_0) & (r_0 \text{ is even}) \\ max\{v_a(p) \mid end(p) = A_0\} - w(A_0) & (r_0 \text{ is odd}) \end{cases}$$

The value $str_a(A_0)$ is defined as the strength of the a-bundle.

Example 7. (Cont'd) In Figure 4, there is one a-bundle $S = \{\langle e, f, g, h\rangle, \langle e, j, k\rangle\}$ and its key is e. Since the number of edges between the root node and e is 4, the strength of e is the sum of the values of the two paths plus the weight of e.
$str_a(e) = \big(v_a(\langle e, f, g, h\rangle) + v_a(\langle e, j, k\rangle)\big) + w(e) = 1/2 + 1 = 3/2.$

The evaluation of the root node in the mWBAF proceeds as follows. For an a-bundle in mWBAF, compute the strength v of the key. Replace the a-bundle by a new node with the weight v, and also re-connect the attack relations between the new node and the nodes outside of the a-bundle to get a reduced mWBAF. Repeat this process until there is only one node. As a result, the strength of the root node is obtained. The details are shown in Appendix.

3.5 Example of hybrid reasoning using a WBAF

A hybrid reasoning using a WBAF proceeds as follows.

> Assume that a litigator suffered a loss of 50,000,000 yen, and is aware of the fact that her opponent's behavior causes this damage. Thus she seeks a law for claiming her damages.

Figure 5 shows a WBAF corresponding to the possible applications of simplified laws regarding claims for damages. The hatched nodes (C1, C2, C3, C4, E1 and E2) show the facts to be shown in this case. Initially, the fact_for_causality (C2) and the facts 50,000,000_yen_damage (C3) are given. There are two laws A1 and A2 that can be applied.

As for A1, if all of the following three hold,
- the contract is unfulfilled in line with purpose (B1),
- a damage occurs (B2),
- there is a causality between the damage and the opponent's behavior (B3),

then the litigator can claim for the damages based on breach of contract (A1), unless the date of unfulfillment is beyond the prescription of three years (D1).

As for A2, if all of the following three hold,
- the opponent violates the duty of care (B4),
- a damage occurs (B2),
- there is a causality between the damage and the opponent's behavior (B3),

then the litigator can claim for the damages based on tort (A2), unless the date of violation is beyond the prescription of ten years (D2).

In this case, first, the arguments B2 and B3 can be derived as conclusions from the facts C2 and C3. Then, two possibly effective laws A1 and A2 are found. To apply A1, the user must prove the fact C1. To apply A2, the user must prove the fact C4.

First, assume that the incident occurred one year ago. Then, E1 attacks D1 and E2 attacks D2. If the intrinsic weight of C1 is higher than that of C4, and those of all other nodes are the same, strength of A1 is higher than that of A2. Therefore, it is advantageous to apply A1; the user focuses on proving C1.

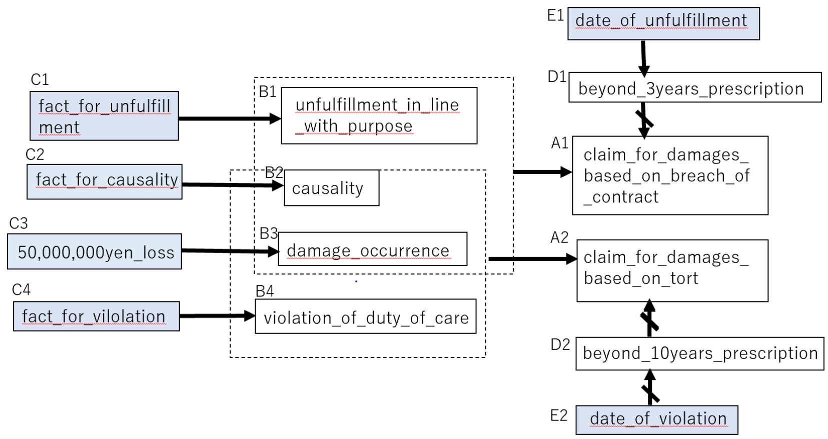

Fig. 5. A WBAF on claim for damages.

However, assume that the incident occurred five years ago. Then, E2 still attacks D2. However, E1 supports D1, and as the result, the strength of A1 becomes negative and rejected. And in this case, the user focuses on proving C4.

This example shows that choice of more effective law depends on the strength of the evidences.

4 Related Works

Several studies have explored the computation of strength and the acceptability of arguments on weighted argumentation frameworks.

Amgoud and Ben-Naim published a series of works regarding the semantics of weighted argumentation frameworks [2–5]. They developed several principles that semantics should satisfy, then proposed a new semantics that satisfied all of those principles. Although some of the fundamental principles they proposed are also satisfied by our method, we cannot adopt them unchanged for the following reasons. In the method of Amgoud and Ben-Naim, the overall strength of a node is computed by aggregating the values of direct supporters and attackers in a breadth-first manner from leaf nodes. The effects of indirect supporters and attackers are gradually propagated to the upper nodes. Conversely, in our method, the strength of a node is computed by surveying all paths in an incremental depth-first manner.

In addition to the works of Amgoud and Naim, several methods have been proposed for defining the strengths of arguments.

An earlier work regarding weighted argumentation frameworks by Dunne et al. [14] assigned a weight to each attack, defined a threshold value, and considered that a set of arguments attacked with a strength under the threshold was acceptable. Furthermore, Dunne et al. developed a method for computation of acceptance and investigated the complexity of this method, but they did not focus on the intrinsic strength of each argument.

Baroni et al. used an acyclic WBAF to develop an original decision-support system QuAD [8], and a later version of this system, termed DF-QuAD [17]. Similar to the approach used by Amgoud and Naim, Baroni et al. defined the strengths of arguments using aggregations of connected arguments in a breadth-first manner.

An attack from collective arguments on an weighted argumentation framework was introduced in several works [6, 15], but they did not deal with a support relation.

Cayrol et al. discussed a support from collective arguments in a bipolar argumentation framework [12]; they did not consider a weight, but their idea of creating a meta-AF by collecting arguments in the same side of for/against the claim initially given is similar to our approach. Furthermore, Cayrol et al. defined a semantics on the meta-AF and found that some properties of the Dung's AF were preserved, while others were lost. They qualitatively considered argument acceptance (accepted or not), whereas we consider the quantitative strengths of collective arguments.

Brewka et al. proposed an abstract dialectical framework (i.e., a generalization of an abstract framework) that managed any relation over a set of arguments [10]. The acceptance condition was represented as a formula for each argument. This was extended to a weighted version [11]. The extent of acceptance of each argument was defined by partial ordering, and several semantics were discussed. However, the objective was not to determine the strength of an argument.

5 Conclusions

We formalized the method of evaluating arguments suitable for a legal simulation, and constructed a system that suggests the most advantageous path to take when seeking to persuade an opponent using WBAF.

We evaluate support and attack relations separately. To evaluate a specific argument, we consider a tree with the argument as its root node. We combine arguments connected by support relations and compute their total strengths; this becomes a meta-argument. Then, we form a meta-WBAF lacking a support relation. We next compute the strength of the root node in the meta-WBAF.

This method is an extension of several existing evaluations but tailored for legal simulation; it has three characteristics: (i) it manages a set-support that reflects the definition of law, (ii) an argument does not always lose strength when it is defended, (iii) the strength of an attack relation is determined by reference to the collective arguments connected in a support relation, rather than by reference to the attacked individual argument. These are essential in a legal simulation.

We applied the method to the hybrid reasoning on WBAF so that it can support a user to find a better solution, and implemented the system using Haskell [19]. The method can be applied to a general decision support system.

In future work, we will extend our method by relaxing the constraints on the WBAF, including a cycle of arguments.

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Appendix.

Reduction of the mWBAF

We reduce mWBAF by recursively shrinking each a-bundle.

Let $mWBAF = \langle AR, ATT, \emptyset, w\rangle$ be an mWBAF, S be an a-bundle of value W, and S' be a new argument. We replace S by a reduced argument S' of weight W. Next, an attack from the key of S is changed to an attack from S'.

[Reduction of a-bundle S (RED(S))]
Let $mWBAF = \langle AR, ATT, \emptyset, w\rangle$ be an mWBAF and A_0 be key of a-bundle S. Let X be $\cup_{p \in S}\{A|A \in p\}$ (i.e., a set of all nodes in S) and S' be a new argument. In addition, we define INR and $OutX$ as follows:

$$INR = \{(A, B)|(A, B) \in ATT, A \in X \wedge B \in X\}$$
$$OutX = \{(S', B)|(A, B) \in ATT, A \in X \wedge B \notin X)\}$$

Then, we perform the following:

- $AR' = AR - X \cup \{S'\}$
  (Replace X by a new argument S'.)
- $ATT' = ATT - INR \cup OutX$
  (Delete the attack relations between the nodes in X and modify the attack relations from/to the nodes in X to a relation from/to S'.)
- $w'(S') = str_a(A_0)$, and $w'(A) = w(A)$ if $A \in AR' \setminus \{S'\}$
  (Set the weight of the new node.)

The results provide a new mWBAF $\langle AR', ATT', \emptyset, w'\rangle$.

We repeat the procedure for each a-bundle until the reduced mWBAF features only the root node.

Example 8. (Cont'd) For $WBAF_2$, we first execute $RED(\{\langle e, f, g, h\rangle, \langle e, j, k\rangle\})$ which creates a new node E for the a-bundle. The strength of E is 3/2 from Example 7 (Figure 6(a)).

There are two maximal attack paths to d: $\langle d, E\rangle$ and $\langle d, i\rangle$. We execute $RED(\{\langle d, E\rangle, \langle d, i\rangle\})$. We compute the value of each maximal attack path to d.

$v_a(\langle d, E\rangle) = \frac{3/2}{\lceil 1/2 \rceil} = 3/2$. $\quad v_a(\langle d, i\rangle) = \frac{1}{\lceil 1/2 \rceil} = 1$.

Note that the number of edges from the root node to the key of a-bundle is unchanged after reduction. Because the number of edges between the root node a and d is 3, the strength of d is the maximum value of the two maximal attack paths minus its own weight. Therefore,

$str_a(d) = max\big(v_a(\langle d, E\rangle), v_a(\langle d, i\rangle\big) - w(d) = max\big(3/2, 1\big) - 1 = 1/2.$

We substitute a new node D for the a-bundle $\{\langle d, E\rangle, \langle d, i\rangle\}$ and obtain the reduced mWBAF (Figure 6(b)).

There is one maximal attack path to a: $\langle a, b, c, D\rangle$. We execute $RED(\{\langle a, b, c, D\rangle\})$. We compute the value of the maximal attack path to a.

$v_a(\langle a, b, c, D\rangle) = -\frac{1}{\lceil 1/2\rceil} + \frac{1}{\lceil 2/2\rceil} - \frac{1/2}{\lceil 3/2\rceil} = -1/4.$

Because the number of edges between the root node a and a is 0, the strength of a is the value of the maximal attack path plus its own weight. Therefore,

$str_a(a) = v_a(\langle a, b, c, D\rangle) + w(a) = -1/4 + 1 = 3/4.$

We substitute a new node A for the a-bundle $\{\langle a, b, c, D\rangle\}$ and obtain the reduced mWBAF (Figure 6(c)).

Because there is now only one root node $\{A\}$, the procedure terminates. Finally, we obtain a value for A of 3/4. This shows that the node a included in A is weakened by argumentation, because $w(a) = 1$.

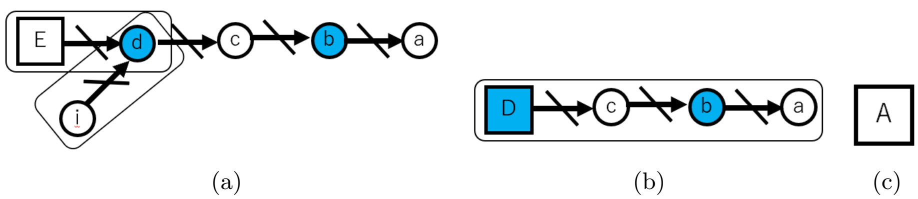

Fig. 6. Reduction of mWBAFs.