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Rectangle Reasoning: A Qualitative Spatial Reasoning with Superposition

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1. Introduction

Qualitative spatial reasoning (QSR) is a method for treating images or figures qualitatively by extracting only the information required by a user for a specified purpose (Aliello et al. 2007). It is widely considered a promising method for reducing memory and workspace requirements for computations that do not involve strict data. However, few studies have developed practical applications. We propose rectangle reasoning as a framework for the application of QSR through an autonomic window placement system. A window can be considered as spatial data of rectangular shape, with changeable size and ratio of edges. Moreover, multiple windows can be displayed in a superposed manner. If we place multiple windows, such that the relevant parts of each are visible and the unnecessary parts are not visible, it provides a display in the most useful form to the user, effectively using a monitor of limited size.

The rectangle reasoning proposed here addresses the relative positional relationships of rectangles with superposition. Each rectangle is represented symbolically using two simple objects, regions and lines, and their relationships, as well as visibility.

In this study, we formalize rectangle reasoning and propose a reasoning algorithm. This algorithm determines whether a figure exists that satisfies all the given visibility requirements in a two-dimensional (2D) plane with foreground/background, and, if such a figure is determined to exist, derives the superposed locations of the rectangles.

This paper is organized as follows. In section 2 we formalize rectangle reasoning; in section 3 we describe the reasoning algorithm; and in section 4 we show our conclusions.

2. Formalization

We describe the structure of our target space intuitively (Figure 1). There is an infinite number of xy-planes in xyz-space that are orthogonal to the z-axis, and their foreground/background relationships are determined by their z-coordinates. For a pair of points with the same xy-coordinates in different 2D planes, the point in the foreground plane hides the point in the background plane. For a finite number of points with the same xy-coordinates in different 2D planes, only the point in the most foreground plane is visible. We call this structure *a quasi-3D space* and the reasoning on this structure *rectangle reasoning*. The target figure is one with a finite number of objects drawn in this structure. An object is a rectangle of changeable size and edge ratio, the sides of which are parallel to the xy-axes, and which is drawn in a unique 2D plane.

A rectangle R is a nonempty set of atomic rectangles $r_1, \cdots, r_n$, each pair of which can share only their boundaries. A rectangle either hides all or no part of an atomic rectangle. That is, a rectangle does not hide a subpart of an atomic rectangle.

Figure 1: The image of quasi-3D space and a target figure

The description language consists of the following nine expressions, of which the intuitive meanings are as follows.

$region(V)$: V is a region
$line(U)$: V is a line segment connecting two points
$super(V)$: V is a region including subregions
$sub(V)$: V is a subregion
$partOf(U,V)$: U is a subregion of a region V
$positionOf(U,V,Pos)$: U is a line at the position Pos of a region V
$lapOver(U,V)$: a region U hides a region V
$visible(V)$: a region V is visible
$invisible(V)$: a region V is invisible

The target figure can be described symbolically using these expressions. However, in a given symbolic expression, the corresponding figure in quasi-3D space does not necessarily exist. Its existence is determined based on the following three operations.

The $create$ operation creates a new symbolic expression representing a rectangle that consists of one atomic rectan-

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gle. The $split_i(i = 1, 2, 3)$ operation divides an atomic rectangle into two atomic rectangles: one subregion and several lines depending on i. The $puton_i$ operation places a rectangle in the foreground of i previously visible atomic rectangles. Note that rectangles that are already superposed are not relocated. This operation creates several $lapOver$, and the corresponding visible subregions become invisible.

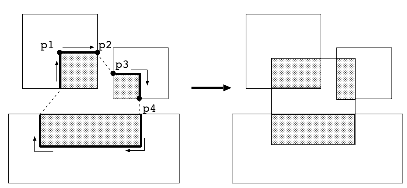
A set of expressions generated from an empty set by finitely applying the operations $create$ and $split_i$ $(i = 1, 2, 3)$ more than once is called a *valid rectangle expression on a plane*. A set of expressions generated from a valid rectangle expression on a plane by finitely applying the operation $puton_i$ is called *a valid rectangle expression*.

Theorem 21. (Kumokawa 2009) (i) For a valid rectangle expression, there exists a target figure in quasi-3D space, (ii) For a target figure in quasi-3D space, there exists a valid rectangle expression.

3. Reasoning on Superposition

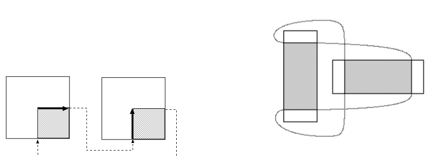
We can derive a location in quasi-3D space of a valid rectangle expression on a plane that guarantees the visibility of certain subregions. We add a *show-requirement* to these subregions as a requirement of visibility.

For a valid rectangle expression, the algorithm searches for a location in which subregions with show-requirements are visible and subregions without show-requirements are invisible. The judgment of whether a superregion can be placed in a specific position is determined by the precondition of $puton$; that is, a superregion can hide a set of subregions simultaneously.



a bold line shows a border and a dotted line shows a supplementary edge

Figure 2: Determining the position of a new rectangle



(a) (b)

Figure 3: Examples in which conditions are not satisfied

Suppose that such a placement is achievable. Here, *a border* is an edge of an atomic rectangle that is not an edge of the rectangle in which it is contained. If all of the atomic rectangles are connected with no space between them, then the area encircled by a circuit of the connected atomic rectangles corresponds to the position of the rectangle. We determine the position of the new rectangle by changing the sizes and ratios of the edges of existing rectangles and atomic rectangles. Otherwise, the borders of the atomic rectangles are traced in the clockwise direction in the following manner. We explain this case using an example.

Consider the case in Figure 2, in which the unshaded atomic rectangles correspond to the subregions with show-requirements, while the shaded ones correspond to the subregions without show-requirements. We show how to judge whether we can place a single rectangle on a given set of atomic rectangles so that the former hides the latter.

Starting at the top line of an atomic rectangle (p1) and reaching the edge of the rectangle in which it is contained (p2), a search is performed for the top line of the border of another atomic rectangle. If such a border can be found (p3), it continues to trace this border until the edge of the rectangle in which it is contained is reached (p4). In this case, *a supplementary edge* is added to connect these points. Otherwise, it searches for the right line, (if there is no right line, then it searches for the bottom line, then the left line) of another atomic rectangle. This procedure is repeated, and terminates if it returns to the initial point. If the traced path obtained in this procedure satisfies the following three conditions, then the area encircled by the path corresponds to the position of the rectangle:

1. All of the borders are contained.
2. Let $p_1, \ldots, p_n$ be a sequence appearing in a path. Then $p_1, \ldots, p_t$ are top edges, $p_{t+1}, \ldots, p_r$ are right edges, $p_{r+1}, \ldots, p_b$ are bottom edges and $p_{b+1}, \ldots, p_n$ are left edges.
3. No pair of borders or supplementary edges cross in the 2D plane.

Then, we determine the position of a new rectangle corresponding to the superregion by changing the sizes and ratios of the edges of the existing rectangles and atomic rectangles.

We can check conditions 1 and 2 by examining the path, and condition 3 by checking whether the numbers of vertices, edges, and faces satisfy Euler's formula when the supplementary edges are added. Figure 3(a) shows an example in which condition 2 is not satisfied, since the sequence of the path is $\{top, left, top, left\}$; Figure 3(b) shows an example in which condition 3 is not satisfied, since supplementary edges cross in a 2D plane.

4. Conclusion

We have demonstrated a rectangle reasoning with superposition. In the future, we will consider the order of arranging the rectangles for efficient placement.

References

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