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Reasoning about Propagation of Properties over Regions

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Abstract: We discuss how a property of some region is propagated to other regions. We propose a system called $SRCC$ that enables the integration of spatial and semantic data. $SRCC$ can represent the relative positions of regions, properties that hold in some regions, semantic relation between regions, and so on. We define the model and describe an algorithm that checks for the existence of a model for a given set of formulas based on this model. We prove the soundness and completeness of the algorithm and apply it to an example that inspects the causality of contamination in 2D space.

Key Words: qualitative spatial reasoning, RCC, GIS, semantic data
Category: D.3.1, F.4.3

1 Introduction

Geographical Information Systems(GIS) [3][8], are usually used to integrate spatial information, such as the shape, size, position or relative position of regions, and semantic information embedded in these regions. Although there have been lots of investigations of the temporal-spatial reasoning mechanisms on GIS, few studies have formalized these mechanisms. In contract, there have been many studies of qualitative spatial reasoning in the field of AI as a formalization of spatial reasoning [1][6][18][21] [22][23]. However, few of these studies handled spatial data incorporating embedded semantic data.

In this paper, we propose a system called $SRCC$ that treats not only spatial relations, but also properties that hold in some regions and semantic relations among regions. In this method, a formula is either in the form of $a\ spatial\ formula$, $a\ property\ formula$ or $a\ propagation\ formula$. Spatial formulas are defined based on RCC [4][5][16]. Properties that hold in some regions are represented by property formulas. The semantic properties relating disconnected regions and the way that the properties of one region affect those of another can be handled using propagation formulas.

We define a model structure and give an algorithm to check the unsatisfiability of a given set of formulas. The algorithm checks the consistency of each region, as well as spatial consistency, that is, the existence of a topological structure that satisfies the spatial part. We apply the algorithm to an example that inspects the causality of contamination in 2D space.

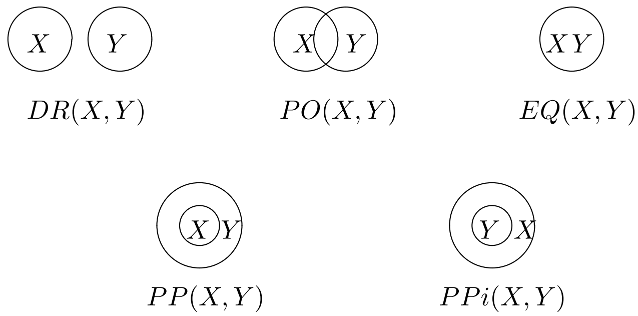

Figure 1: The basic relations of RCC-5

We also present an algorithm to derive the spatial constraints that are not explicitly specified and apply it to determine the place where a new region is introduced.

Furthermore, we discuss the different types of semantic properties of the region: properties that hold in a region and properties that hold with respect to a region $as\ a\ whole$.

This paper is organized as follows. First, we explain $SRCC$, giving the definition of the description language and its semantics in section 2. Then, we present a checking algorithm in section 3, and show applications in section 4 and 5. We discuss the approach in section 6 and finally we conclude in section 7.

2 $SRCC$

Region Connection Calculus(RCC) [4][5][16] is one of the representatives of theories that consider a space as a set of regions, paying attention only to their relative positions, and that provide qualitative spatial representation and reasoning. Figure 1 shows the basic relations of RCC-5[1], a variant of RCC. These relations are jointly exhaustive and pairwise disjoint. We propose a system $SRCC$ that extends RCC-5 so that it can handle the spatial data incorporated with semantic data.

2.1 Description Language

The description language of $SRCC$ is defined as follows.

1. region term

   (a) A region variable (denoted $X, Y, Z, \ldots$) is a term.

[1] In RCC-5, the boundaries of regions are not taken into account, which differs from RCC-8, another variant of RCC.

   (b) $f(\alpha)$ is a region term, where f is a function symbol and α is a region term[2].

2. formula
   (a) spatial formula
   $DR(\alpha, \beta)$, $PO(\alpha, \beta)$, $EQ(\alpha, \beta)$, $PP(\alpha, \beta)$, $PPi(\alpha, \beta)$ are spatial formulas, where α and β are region terms. The formulas constructed from these using Boolean operators in the usual way are spatial formulas.

   (b) property formula
   $[\Box\alpha, G]$ are $[\Diamond\alpha, G]$ are property formulas, where α is a region term and G is a literal (of propositional logic). The formulas constructed from these using Boolean operators in the usual way are property formulas.

   (c) propagation formula
   $[*\alpha, G] \wedge \theta \Rightarrow [*\beta, H]$ is a propagation formula, where G and H are literals (of propositional logic), θ (sometimes not appear) is a spatial formula on α and β ($*\alpha$ denotes either $\Box\alpha$ or $\Diamond\alpha$, and we use this notation hereafter).

We use the spatial formula $P(\alpha, \beta)$ to denote the part-of relation defined by $PP(\alpha, \beta) \vee EQ(\alpha, \beta)$, and $Pi(\alpha, \beta)$ to denote $PP(\beta, \alpha)$.

$[\Box\alpha, G]$ indicates that G holds everywhere in α, whereas $[\Diamond\alpha, G]$ indicates that G holds somewhere in α.

Spatial formulas show the relations over regions. Property formulas show the properties that hold in a region. Propagation formulas show the relation between the properties of some regions and those of other regions. Propagation occurs only when the condition θ is satisfied. If $G = H$, then it shows that a property that holds in α is propagated to β, otherwise, the property is changed.

These are two types of propagation: functional propagation and positional propagation.

Functional propagation is available only for regions satisfying the specified semantic relations. They are explicitly described in the form: $[*\alpha, G] \wedge \theta \Rightarrow [*f(\alpha), H]$. Conversely, positional propagation is available for all regions satisfying some spatial relation. The following propagation formulas show positional propagation that holds for any α, β and G.

Ax1 $[\Box\alpha, G] \Leftrightarrow \neg[\Diamond\alpha, \neg G]$

Ax2 $[\Box\alpha, G] \Rightarrow [\Diamond\alpha, G]$

Ax3 $[\Box\beta, G] \wedge P(\alpha, \beta) \Rightarrow [\Box\alpha, G]$

Ax4 $[\Box\alpha, G] \wedge P(\alpha, \beta) \Rightarrow [\Diamond\beta, G]$

[2] For simplicity, we use unary function symbols here, but we can extend the language by allowing n-ary function symbols.

Ax5 $[\Box\alpha, G] \wedge PO(\alpha, \beta) \Rightarrow [\Diamond\beta, G]$

2.2 The Model

We call a regular subset of a topological space $a\ region$. A region may be multipiece or may contain a hole, but it must consist of a universal dimension.

Definition 1.

For a set of spatial formulas, if there exists a topological space that realizes all the formulas, then the set is said to be *RCC-satisfiable*.

Definition 2.

We define *a structure* $\mathcal{M} = \langle \mathbf{D}, \Sigma, \Phi \rangle$ as follows.

$\mathbf{D}$ is a set of regions, Σ is a set of spatial formulas, and Φ is the set $\{[\Box X_1, G_1], \ldots, [\Box X_n, G_n]\}$, where $X_1, \ldots, X_n$ are region variables that appear in Σ, and $G_1, \ldots, G_n$ are literals of propositional logic. For region variable X that appears in Σ, we assign an element of $\mathbf{D}$, and for a function symbol f, we assign a map $\mathbf{D} \to \mathbf{D}$. Moreover, $\mathcal{M}$ should satisfy the following conditions (i)$\sim$(iii).

(i) There exists a topological space $\mathcal{T}$ that realizes all the formulas in Σ [3].

(ii) For any X and G, $[\Box X, G] \in \Phi$ and $[\Box X, \neg G] \in \Phi$ do not hold at the same time.

(iii) If $P(\alpha, \beta)$ is realized on $\mathcal{T}$ and $[\Box\beta, G] \in \Phi$ holds, then $[\Box\alpha, G] \in \Phi$ holds.

If neither $[\Box X, G] \in \Phi$ nor $[\Box X, \neg G] \in \Phi$ holds, then it means that G holds in some part of X, and $\neg G$ holds in another part of X.

When a formula φ is true in structure $\mathcal{M}$, we denote this by $\mathcal{M} \models \varphi$. The semantics is defined as follows.

1. $\mathcal{M} \models \varphi$ iff $\varphi \in \Sigma$ where φ is a spatial formula.
2. $\mathcal{M} \models [\Box X_i, G_i]$ iff $[\Box X_i, G_i] \in \Phi$
3. $\mathcal{M} \models \neg\varphi$ iff not $\mathcal{M} \models \varphi$.
4. $\mathcal{M} \models \varphi \wedge \psi$ iff $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
5. $\mathcal{M} \models \varphi \vee \psi$ iff $\mathcal{M} \models \varphi$ or $\mathcal{M} \models \psi$
6. $\mathcal{M} \models \varphi \Rightarrow \psi$ iff not $\mathcal{M} \models \varphi$ or $\mathcal{M} \models \psi$
7. $\mathcal{M} \models \varphi \Leftrightarrow \psi$ iff both $\mathcal{M} \models \varphi$ if and only if $\mathcal{M} \models \psi$

[3] We do not discuss here how to decide the existence of such a topological space. This is explained, for example, in [23].

Definition 3.

Let φ be a formula. If there exists a structure $\mathcal{M}$ such that $\mathcal{M} \models \varphi$, then it is said that φ is $satisfiable$, and $\mathcal{M}$ is said to be ***a model*** for φ. Otherwise, φ is said to be $unsatisfiable$.

Let $\Lambda = \{\varphi_1, \ldots, \varphi_n\}$ be a set of formulas and ψ be a formula. For every model $\mathcal{M}$ in which $\varphi_1 \wedge \ldots \wedge \varphi_n$ is satisfiable, if ψ is satisfiable in $\mathcal{M}$, then ψ is said to be $a\ logical\ consequence$ of Λ, and it is denoted by $\Lambda \vdash \psi$.

Theorem 4.

Let Λ be a set of formulas and ψ be a formula. $\Lambda \cup \{\neg\psi\}$ is unsatisfiable if and only if $\Lambda \vdash \psi$.

Proof)

Similar to the case in propositional logic.

3 Reasoning Based on $SRCC$

3.1 Checking Algorithm

For a given finite set of formulas Λ, we present an algorithm for checking whether it is unsatisfiable.

In this algorithm, we check the consistency of each region, as well as spatial consistency, that is, the existence of a topological structure that realizes the spatial formulas.

<u>Checking Algorithm</u>

Let Θ and Δ be a set of spatial formulas and property formulas appearing in Λ, respectively.

1. Set $i = 0$.

   Let $\Delta_0 = \Delta$ and $\lambda_0 = \Delta$.

2. (RCC-satisfiability check)

   If there exists no topological space that realizes all the formulas of Θ, then Λ is unsatisfiable. Otherwise, continue.

3. (consistency check of each region)

   If there exists a region α and property formulas $\delta_{i1}, \ldots, \delta_{in} \in \Delta_i$ such that $\delta_{ij}(j = 1, \ldots, n)$ is $[\alpha_i, G_{ij}]$ where α_i is either $\Box\alpha$ or $\Diamond\alpha$, and $\delta_{i1} \wedge \ldots \wedge \delta_{in} \vdash \bot$, then Λ is unsatisfiable. Otherwise, continue.

4. Do the following:

(a) For each propagation formula $\delta \wedge \theta \Rightarrow \delta'$ in Λ, if $\Delta_i \vdash \delta$ and $\Theta \vdash \theta$, then δ' is regarded as an additional formula and the propagation formula is extracted from Λ.

(b) For each property formula δ in λ_i, if either of axioms $Ax2 \sim Ax5$ is applicable, then the right-hand-side of the resulting formula is regarded as an additional formula and let it be δ'.

(c) Set λ_{i+1} be a set of additional formulas. If $\lambda_{i+1} = \{\}$, then Λ is satisfiable. Otherwise, set $\Delta_{i+1} = \Delta_i \cup \lambda_{i+1}$.

5. Set $i = i + 1$ and go to 3.

In process 3, the unsatisfiability of region α is checked, which is reduced to that of propositional logic.

3.2 Soundness and Completeness

First, we prove that the checking algorithm is sound.

Lemma 5.

Let Λ be a finite set of formulas and let Δ_i be the set of formulas defined in the checking algorithm for Λ.

For any property formula $\delta \in \Delta_i$, $\Lambda \vdash \delta$ holds.

Proof)

We prove this lemma by induction.

1. Assume that $i = 0$.

   For $\delta \in \Delta_0 (= \Delta)$, since $\Delta \vdash \delta$ and $\Delta \subseteq \Lambda$, $\Lambda \vdash \delta$ holds.

2. Assume that if the lemma holds for Δ_i. Δ_{i+1} is generated by adding a set of additional formulas to Δ_i.

   (Case1) δ' is added by the process 4(a).

   Let the corresponding propagation formula be $\delta \wedge \theta \Rightarrow \delta'$. From $\Delta_i \vdash \delta$, there exist $\delta_1, \ldots, \delta_n \in \Delta_i$ such that $\delta_1 \wedge \ldots \wedge \delta_n \vdash \delta$. From induction hypothesis, $\Lambda \vdash \delta_1 \wedge \ldots \wedge \delta_n$. Therefore, $\Lambda \vdash \delta$ holds. From $\Theta \vdash \theta$ and $\Theta \subseteq \Lambda$, $\Lambda \vdash \theta$ holds. Thus, $\Lambda \vdash \delta'$ holds.

   (Case2) δ' is added by the process 4(b).

   δ' is obtained as a result of applying any of $Ax2 \sim Ax5$. In this case, $\Lambda \vdash \delta'$ holds trivially.

Q.E.D.

Theorem 6.

In the checking algorithm, if either 1 or 2 in the following holds, then Λ is unsatisfiable.

1. Θ is not RCC-satisfiable.
2. There exists a region α and property formulas $\delta_{i1}, \ldots, \delta_{in} \in \Delta_i$ such that $\delta_{ij}(j = 1, \ldots, n)$ is $[\alpha_i, G_{ij}]$ where α_i is either $\Box\alpha$ or $\Diamond\alpha$, and $\delta_{i1} \wedge \ldots \wedge \delta_{in} \vdash \bot$.

Proof)

1. Trivial.
2. $\Lambda \vdash \delta_{i1} \wedge \ldots \wedge \delta_{in}$ holds from lemma 5. Thus, $\Lambda \vdash \bot$ holds.

Theorem 7.

The checking algorithm terminates in a finite time.

Proof)

Although the function symbol over a region term is introduced, the number of regions is finite according to the syntactic definition. Moreover, the regions not appearing in Λ are not newly created in the checking algorithm. The consistency check for each region in the checking algorithm is reduced to that of propositional logic. Therefore, the checking algorithm terminates in a finite time.

Furthermore, checking algorithm is complete.

Theorem 8.

If Λ is unsatisfiable, then the checking algorithm terminates with the answer "unsatisfiable."

Proof)

Theorem 7 guarantees the termination of the checking algorithm.

Assume that the algorithm terminates with the answer "satisfiable." Then we have the set $\Delta_k \cup \Theta_k$, at termination. We examine three conditions for a model in Definition 2 to prove that $\Delta_k \cup \Theta_k$ is a model.

(i) There exists a topological space that realizes Θ_k, since its RCC-satisfiability is checked.

(ii) For any X and G, $[\Box X, G] \in \Delta_k$ and $[\Box X, \neg G] \in \Delta_k$ do not hold at the same time, since the consistency of each region is checked.

(iii) For a pair of regions that satisfy $P(\alpha, \beta)$, if $[\Box\beta, G] \in \Delta_k$, then $[\Box\alpha, G] \in \Delta_k$ holds since $[\Box\alpha, G]$ must be added by applying $Ax3$.

Therefore, since all the conditions are satisfied, $\Delta_k \cup \Theta_k$ is a model. Thus, there exists a model, which is a contradiction.

Hence, the theorem holds.

Q.E.D.

(a) (b)

Figure 2: Inspection of the causality of contamination

3.3 Reasoning over Regions

We propose two reasoning algorithms over regions using the checking algorithm. One algorithm proves whether some property ψ holds under the conditions $\varphi_1, \ldots, \varphi_n$. We apply the checking algorithm to prove the unsatisfiability of $\{\varphi_1, \ldots, \varphi_n, \neg\psi\}$ and the satisfiability of $\{\varphi_1, \ldots, \varphi_n\}$. The other one derives the spatial constraint that makes a set of formulas $\{\varphi_1, \ldots, \varphi_n\}$ (un)satisfiable. In the following two sections, we describe these algorithms using examples that inspects the causality of contamination in 2D space.

4 Application – Proving the Property

First, we prove that properties of one region affect those of another region.

$Example\ 1.$

Consider a river with a metal manufacturing factory in an upstream area, and a residental area downstream. If the factory contaminates its environment, are chemicals detected in the residental area?

We describe this problem in $SRCC$. mtl, ur, dr and res are region variables that denote the metal manufacturing factory, the area upstream of the river, the area downstream of the river, and the residents, respectively. The propositions $Contam$ and $Chem$ show the property of being contaminated by the factory and that chemicals are detected, respectively. The function $flow$ maps the region from the upstream portion of the river to the region to which the river flows. For simplicity, we assume there are no effect unless explicitly represented.

Then, the problem is formalized as follows:

$\varphi_1 : PP(mtl, ur)$
$\varphi_2 : PP(res, dr)$
$\varphi_3 : EQ(flow(ur), dr)$
$\varphi_4 : [\Box mtl, Contam]$
$\varphi_5 : [\Diamond ur, Contam] \Rightarrow [\Box flow(ur), Chem]$
$\varphi_6 : [\Box mtl, Contam] \Rightarrow [\Box mtl, Chem]$

The conclusion is represented as follows:

$\psi : [\Diamond res, Chem]$

and its negation is:

$\neg\psi : [\Box res, \neg Chem]$

Instead of proving $\varphi_1 \wedge \ldots \wedge \varphi_6 \vdash \psi$ directly, we prove the unsatisfiability of $\{\varphi_1, \ldots, \varphi_6\} \cup \{\neg\psi\}$ using the checking algorithm. In this example, $\Lambda = \{\varphi_1, \ldots, \varphi_6, \neg\psi\}$, $\Theta = \{\varphi_1, \varphi_2, \varphi_3\}$ and $\Delta = \{\varphi_4, \neg\psi\}$.

Step1:

$\Delta_0 = \{[\Box mtl, Contam], [\Box res, \neg Chem]\}$.

$\Theta_0 = \{PP(mtl, ur), PP(res, dr), EQ(flow(ur), dr)\}$.

Step2:

There exists a 2D space that realizes Θ, such as that shown in Figure 2(a). Therefore, Θ is RCC-satisfiable.

Then, we check the consistency of each region. Since neither $[\Box mtl, Contam] \vdash \bot$ nor $[\Box res, \neg Chem] \vdash \bot$ holds, we go to 4 in the checking algorithm.

From φ_6, $[\Box mtl, Chem]$ is an additional formula.

From $Ax2$, $[\Diamond mtl, Contam]$ is an additional formula.

From $Ax2$, $[\Diamond res, \neg Chem]$ is an additional formula.

As a result, $\Delta_1 = \Delta_0 \cup \{[\Box mtl, Chem], [\Diamond mtl, Contam], [\Diamond res, \neg Chem]\}$.

Step3:

From $Ax2$, $[\Diamond mtl, Chem]$ is an additional formula.

From φ_1 and $Ax4$, $[\Diamond ur, Contam]$ is an additional formula.

From φ_2 and $Ax4$, $[\Diamond dr, \neg Chem]$ is an additional formula.

As a result, $\Delta_2 = \Delta_1 \cup \{[\Diamond mtl, Chem], [\Diamond ur, Contam], [\Diamond dr, \neg Chem]\}$.

Step4:

From φ_5, $[\Box flow(ur), Chem]$ is an additional formula.

From φ_1 and $Ax4$, $[\Diamond ur, Chem]$ is an additional formula.

As a result, $\Delta_3 = \Delta_2 \cup \{[\Box flow(ur), Chem], [\Diamond ur, Chem]\}$.

Step5:

From $Ax2$, $[\Diamond flow(ur), Chem]$ is an additional formula.

From φ_3 and $Ax3$, $[\Box dr, Chem]$ is an additional formula.

As a result, $\Delta_4 = \Delta_3 \cup \{[\Diamond flow(ur), Chem], [\Box dr, Chem]\}$.

Step6:

Both $[\Diamond dr, \neg Chem]$ and $[\Box dr, Chem]$ are contained in Δ_4. $[\Diamond dr, \neg Chem] \wedge [\Box dr, Chem] \vdash \bot$ holds. Hence, $\{\varphi_1, \ldots, \varphi_6\} \cup \{\neg\psi\}$ is unsatisfiable.

Note that if $\varphi_1 \wedge \ldots \wedge \varphi_6 \vdash \bot$, then we can take any formula as ψ. Therefore, we must examine the satisfiability of $\varphi_1 \wedge \ldots \wedge \varphi_6$. Since all the propositions appearing in $\varphi_1 \wedge \ldots \wedge \varphi_6$ are positive (i.e. no negation appears), there exist no property formulas $\delta_1, \ldots, \delta_n$ such that $\delta_1 \wedge \ldots \wedge \delta_n \vdash \bot$ in any region. Therefore, it is easy to prove satisfiability.

$Example$ 2.

Given the same condition as in Example 1, assume that we will build a filtration plant. We prove that it prevents chemicals from reaching the residents. Here, $filter$ is a region variable that denotes the filtration plant.

Then, the problem is formalized as follows:

$\varphi_1 : PP(mtl, ur)$
$\varphi_2 : PP(res, dr)$
$\varphi_3 : EQ(flow(ur), filter)$
$\varphi_4 : EQ(flow(filter), dr)$
$\varphi_5 : DR(filter, flow(filter))$
$\varphi_6 : [\Box mtl, Contam]$
$\varphi_7 : [\Diamond ur, Contam] \Rightarrow [\Box flow(ur), Chem]$
$\varphi_8 : [\Diamond filter, Chem] \wedge DR(filter, flow(filter)) \Rightarrow [\Box flow(filter), \neg Chem]$
$\varphi_9 : [\Box mtl, Contam] \Rightarrow [\Box mtl, Chem]$

The conclusion is represented as follows:

$$\psi : [\Box res, \neg Chem]$$

and its negation is:

$$\neg\psi : [\Diamond res, Chem]$$

We prove the unsatisfiability of $\{\varphi_1, \ldots, \varphi_9\} \cup \{\neg\psi\}$. The proof proceeds according to the checking algorithm, where $\Lambda = \{\varphi_1, \ldots, \varphi_9, \neg\psi\}$, $\Theta = \{\varphi_1, \ldots, \varphi_5\}$ and $\Delta = \{\varphi_6, \neg\psi\}$.

There exists a 2D space that realizes Θ, such as that shown in Figure 2(b). Therefore, Θ is RCC-satisfiable.

Then, we check consistency of each region.

Using a process similar to that in Example 1, we obtain:

$\Delta_0 = \{[\Box mtl, Contam], [\Diamond res, Chem]\}$.
$\Delta_1 = \Delta_0 \cup \{[\Box mtl, Chem], [\Diamond mtl, Contam], [\Diamond dr, Chem], \}$.
$\Delta_2 = \Delta_1 \cup \{[\Diamond mtl, Chem], [\Diamond ur, Contam]\}$.
$\Delta_3 = \Delta_2 \cup \{[\Box flow(ur), Chem], [\Diamond ur, Chem]\}$.
$\Delta_4 = \Delta_3 \cup \{[\Box filter, Chem], [\Diamond flow(ur), Chem]\}$.

$\Delta_5 = \Delta_4 \cup \{[\Diamond filter, Chem]\}$.
$\Delta_6 = \Delta_5 \cup \{[\Box flow(filter), \neg Chem]\}$.
$\Delta_7 = \Delta_6 \cup \{[\Diamond flow(filter), \neg Chem], [\Box dr, \neg Chem]\}$.

In Δ_7, $[\Box dr, \neg Chem] \wedge [\Diamond dr, Chem] \vdash \perp$ holds. Hence, $\{\varphi_1, \ldots, \varphi_9\} \cup \{\neg\psi\}$ is unsatisfiable.

Next, we examine the satisfiability of $\varphi_1 \wedge \ldots \wedge \varphi_9$.

$\Delta'_0 = \{[\Box mtl, Contam]\}$.

The check proceeds similarly, generating $\Delta'_1, \Delta'_2, \ldots$, and we obtain:

$\Delta'_7 = \Delta_7 - \{[\Diamond res, Chem], [\Diamond dr, Chem]\}$.
$\Delta'_8 = \Delta'_7 \cup \{[\Diamond dr, \neg Chem], [\Box res, \neg Chem]\}$.
$\Delta'_9 = \Delta'_8 \cup \{[\Diamond res, \neg Chem]\}$.

Since no more propagation formulas can be applied, the formula is proved to be satisfiable.

5 Application – Deriving Constraints on the Regions

In Example 2, when the new region $filter$ is introduced, both the spatial constraint (φ_5) and the characteristics of propagation (φ_8) are clear. However, these factors are sometimes unclear. In this section, we discuss how to derive the spatial constraint on a newly introduced region.

Furthermore, we apply the checking algorithm to check the satisfiability of $\{\varphi_1, \ldots, \varphi_n\}$ to prove $\varphi_1 \wedge \ldots \wedge \varphi_n \vdash \psi$. Note that $\varphi_1, \ldots, \varphi_n, \neg\psi$ does not include all the relations of every pair of regions. That is, there are no constraints for some pair of regions. If we add some spatial relation φ, then $\{\varphi_1, \ldots, \varphi_n, \varphi\}$ may become unsatisfiable. For instance, there is no problem if $PP(ur, dr)$ holds in Example 1, while it *is* a problem if $PO(mtl, res)$ holds in Example 2. In this section, we also discuss an algorithm to derive the spatial constraints that are not explicitly specified. We examine the conditions determining where to place the filtration plant and detect the spatial constraint that prevents the chemicals from entering the residental region in Example 2.

5.1 The Spatial Constraints on Newly-Introduced Regions

We explain the procedure using Example 2.

Example 3.

Assume that φ_5 does not hold, that is, the site of the filtration plant is not clear.

Then, $DR(filter, flow(filter))$ in φ_8 does not hold, that is, φ_8 cannot be applied. Thus, $\varphi_1 \wedge \ldots \wedge \varphi_4 \wedge \varphi_6 \wedge \ldots \wedge \varphi_9 \wedge \neg\psi$ is satisfiable. It follows that φ_5 indicates a constraint that $filter$ and $flow(filter)$ which is equivalent to dr should be disconnected.

It is easy to find the relation φ_5, if φ_8 is given. However, this raises the question of what happens if φ_8 is not given.

We use the following formula to solve this problem.

Theorem 9.

$[\Box\alpha, G] \wedge [\Box\beta, \neg G] \Rightarrow DR(\alpha, \beta)$. $\quad [\Box\alpha, G] \wedge [\Diamond\beta, \neg G] \Rightarrow \neg Pi(\alpha, \beta)$.

These formulas show the constraints on the relation between regions whose properties are inconsistent with each other. It is proved using the axioms in section 2.1.

The following theorem is the generalized version.

Theorem 10.

If $G \wedge H \Rightarrow \bot$, then $[\Box\alpha, G] \wedge [\Box\beta, H] \Rightarrow DR(\alpha, \beta)$ and $[\Box\alpha, G] \wedge [\Diamond\beta, H] \Rightarrow \neg Pi(\alpha, \beta)$.

Example 4.

Assume that φ_5 does not hold, and φ'_8 is given instead of φ_8.

φ'_8: $[\Diamond filter, Chem] \Rightarrow [\Box flow(filter), \neg Chem]$

This means that the function of the filtration is clear, but the spatial constraint is unclear.

In this case, using Theorem 9, $DR(filter, flow(filter))$ is obtained, since both $[\Box flow(filter), \neg Chem]$ and $[\Box filter, Chem]$ are contained in Δ_6.

5.2 Unspecified Relations

Which relations hold between the regions besides the relations that are specified explicitly? Are there any constraints on the filtering plant that will keep chemicals from the residental area in Example 2?

We provide an algorithm for deriving the spatial constraints. For a given set $\{\varphi_1, \ldots, \varphi_n\}$, execute the checking algorithm as far as possible to obtain Δ_k. If there exist both $[\Box\alpha, G]$ and $[\Box\beta, H]$ in Δ_k, where $G \wedge H \Rightarrow \bot$, then $DR(\alpha, \beta)$ must hold.

Theorem 11.

Assume that $G \wedge H \Rightarrow \bot$ holds. Then, $DR(\alpha, \beta)$ if there exists no model that satisfies $[\Box\alpha, G] \wedge [\Box\beta, H]$.

Example 5.

We apply the algorithm to Example 1.

The specified relations between regions are shown in Table 1. Note that $flow(ur)$ is regarded as dr in this table. We determine the constraints on the ANY-part in the table.

We start checking algorithm for $\varphi_1, \ldots, \varphi_6$ as far as possible. Finally we get:

$\alpha \setminus \beta$	ur	mtl	dr	res
ur	EQ	PPi	ANY	ANY
mtl	PP	EQ	ANY	ANY
dr	ANY	ANY	EQ	PPi
res	ANY	ANY	PP	EQ

Table 1: The specified relations $R(\alpha, \beta)$ between regions in Example 1

$$\begin{aligned}\mathit{\Delta}_5 = \{\ & [\Box mtl, Contam], [\Box mtl, Chem], [\Diamond mtl, Contam], [\Diamond mtl, Chem],\\ & [\Diamond ur, Contam], [\Box flow(ur), Chem], [\Diamond ur, Chem], [\Diamond flow(ur), Chem],\\ & [\Box dr, Chem], [\Box res, Chem], [\Diamond dr, Chem], [\Diamond res, Chem]\ \}\end{aligned}$$

Since there is no pair in $\mathit{\Delta}_5$ that satisfies Theorem 10, we do not have to set any constraint. That is, ANY means any relations.

$Example$ 6.

We apply the algorithm to Example 2. The relations between regions are shown in Table 2. Note that $flow(ur), flow(filter)$ are regarded as $filter$ and dr, respectively. We determine the constraints on the ANY-part in the table.

We start checking algorithm for $\varphi_1, \ldots, \varphi_9$ as far as possible. Finally we get:

$$\begin{aligned}\mathit{\Delta}_9 = \{\ & [\Box mtl, Contam], [\Box mtl, Chem], [\Diamond mtl, Contam], [\Diamond mtl, Chem],\\ & [\Diamond ur, Contam], [\Box flow(ur), Chem], [\Diamond ur, Chem], [\Box filter, Chem],\\ & [\Diamond flow(ur), Chem], [\Diamond filter, Chem], [\Box flow(filter), \neg Chem],\\ & [\Diamond flow(filter), \neg Chem], [\Box dr, \neg Chem], [\Diamond dr, \neg Chem], [\Box res, \neg Chem],\\ & [\Diamond res, \neg Chem]\ \}\end{aligned}$$

Since both $[\Box mtl, Chem]$ and $[\Box dr, \neg Chem]$ are contained in $\mathit{\Delta}_9$, $DR(mtl, dr)$ is derived. Similarly, we can derive the relations: $\neg P(ur, dr), DR(mtl, dr)$, $DR(filter, dr), \neg P(ur, res), DR(mtl, res), DR(filter, res)$. The resulting relations are shown in Table 3.

As a result, we found that if mtl and dr share a common part, then the construction of a filtration plant is meaningless. Moreover, we also found that there is no spatial constraints, for example, between ur and $filter$. This means that a filtration plant can be built in the upper part of the river.

6 Discussion

Although there have been lots of work on formal methods of spatial reasoning, few studies have been done on integration of spatial and semantic data.

Eschenbach proposed predication calculus [9] which can handle the semantic properties of regions in addition to their mereological and topological properties by introducing predicators that mean “somewhere in a region” and “everywhere in a region.”

$\alpha \setminus \beta$	ur	mtl	dr	res	$filter$
ur	EQ	PPi	ANY	ANY	ANY
mtl	PP	EQ	ANY	ANY	ANY
dr	ANY	ANY	EQ	PPi	ANY
res	ANY	ANY	PP	EQ	ANY
$filter$	ANY	ANY	DR	ANY	EQ

Table 2: The specified relation $R(\alpha, \beta)$ between regions in Example 2

$\alpha \setminus \beta$	ur	mtl	dr	res	$filter$
ur	EQ	PPi	¬ P	¬ P	ANY
mtl	PP	EQ	DR	DR	DR
dr	¬ Pi	DR	EQ	PPi	DR
res	¬ Pi	DR	PP	EQ	DR
$filter$	ANY	DR	DR	DR	EQ

Table 3: The relation $R(\alpha, \beta)$ between regions in Example 2 - after the constraints are found

Although their paper discussed the representation of semantic properties, no inference system was described. They presented a composition table for reasoning spatial relations, but did not describe any procedures to check validity or unsatisfiability of a set of formulas. On the other hand, we present here an algorithm that checks unsatisfiability.

In this study, we regarded a region as an infinite set of points, and the property of a region as inherited by its proper part. $\Box\alpha$ means that a property holds for every point in α. However, there are different types of property such as the population or size of a region. For example, the fact that the population of city C is 10,000 does not imply that the population of a town in C is 10,000. In this case, $Ax3$ which we used in this study does not hold because the population and size are properties that hold for a region as a whole. It follows that we have to introduce another operator $\triangle$ to describe such alternate properties. The above example is represented as $[\triangle C, 10000]$. $\triangle$, for example, should satisfy the axiom: $\neg[\triangle X, G] \Leftrightarrow [\triangle X, \neg G]$. However, it is not clear whether these two types of properties can be treated in a unified manner, or whether a formal system can be well-defined. An attempt to incorporate size with a spatial relation was presented in [12], which introduced the binary operators $<, =, >$ to handle the size of regions qualitatively.

7 Conclusion

In this paper, we have discussed how a property of some region is propagated to other regions.

We have presented the system $SRCC$ which can treat spatial data with embedded semantic data. We have defined the description language that can represent the relative positions of regions, properties that hold in some regions, semantic relations between regions and so on. We have presented an algorithm to check unsatisfiability for a given set of formulas, and applied it to an example inspecting the causality of contamination in 2D space. We have also presented an algorithm to derive spatial constraints that are not specified explicitly and used it to determine the place where a new region is introduced.

In future, we will explore the theoretical aspects of $SRCC$ more deeply, and discuss the limits of its expressive power. We will also consider integrating temporal reasoning in $SRCC$.

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