Absolutely Secure Message Transmission Using a Key Sharing Graph

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Abstract

Assume that there are players and an eavesdropper Eve of unlimited computational power and that several pairs of players have shared secret keys beforehand. In a key sharing graph, each vertex corresponds to a player, and each edge corresponds to a secret key shared by the two players corresponding to the ends of the edge. Given a key sharing graph, a player wishes to send a message to another player so that the eavesdropper Eve and any other player can get no information on the message. In this thesis, we first give a necessary and sufficient condition on a key sharing graph for the existence of such a unicast protocol. We then extend the condition to the case where a multiple number of players other than the sender and receiver passively collude. We finally give a sufficient condition for the existence of a secure multicast protocol.

1 Summary

Achieving secure communication has been one of the most important problems in information technology. There are a lot of works on secure message transmissions. Many conventional models assume a communication network consisting of various channels: private communication channels, partial broadcast channels [2], multicast channels and neighbor network channels [1]. In contrast, our model assumes that every communication is done through a public network and may be overheard by anyone, but assumes that several pairs of players have shared secret keys beforehand as in [3, 4]. Under such a model we deal with a secure message transmission using a public network and secret keys shared by several pairs of players. More precisely, we consider the following situation.

There are $n (\geq 2)$ players $p_1, p_2, \dots, p_n$ and an eavesdropper Eve of unlimited computational power, and several pairs of players have already shared secret keys. For simplicity, a message and all secret

keys are assumed to be one-bit numbers, although our results can be easily generalized to the case where a message and all keys are ℓ -bit numbers for a positive number ℓ . Let G be a graph, called a *key sharing graph* [3, 4], in which each vertex represents a player p_i , $1 \leq i \leq n$, and each edge $p_i p_j$ joining vertices p_i and p_j represents a secret key $k_{ij} \in \{0, 1\}$ shared by players p_i and p_j , as illustrated in Fig. 1. A secret key k_{ij} is known only to players p_i and p_j . All players and Eve know the shape of graph G . Every communication (conversation) between players through a public network can be overheard by any player and Eve. Since Eve has an unlimited computational power, one cannot use any cryptographic protocol based on the computational hardness assumption such as the RSA public key cryptosystem. All players can generate random numbers (flip coins), and are “honest,” that is, follow a specified protocol and do not “lie.”

Figure 1: An example of key sharing graph G .

Under such a situation, if G is a connected graph, that is, G has a spanning tree as drawn by thick lines in Fig. 1, then any player, say p_1 , can send a one-bit message to all the other players so that the eavesdropper Eve can get no information on the message. Clearly, this can be done, within $n - 1$ communication rounds, by the “flooding protocol”, which floods the message from p_1 to the other players by using the secret keys on a spanning tree. This can also be done with exactly one communication round [4].

In this thesis we first consider a unicast protocol by which a player, say p_1 , sends a message $m \in \{0, 1\}$ only to a designated player, say p_2 , so that Eve and any player other than p_1 and p_2 cannot get any information on m . We prove that there is such a unicast protocol if and only if G has either an edge p_1p_2 joining vertices p_1 and p_2 or a pair of paths between p_1 and p_2 having no common vertices except p_1 and p_2 . This result immediately implies that such a secure message transmission can be done between every pair of players if and only if G is a 2-connected graph.

We then extend the result to the case where a multiple number of players other than p_1 and p_2 passively collude. More precisely, we showed that a player p_1 can send a message m to another player p_2 absolutely securely against a c -collusive attack if and only if the key sharing graph G has either an edge joining p_1 and p_2 or a set of $c + 1$ internally disjoint paths between p_1 and p_2 . This result implies that every player can send a message to any other player absolutely securely against a c -collusive attack if and only if the key sharing graph G is $(c + 1)$ -connected.

We finally give a sufficient condition for the existence of a multicast protocol by which a player p_1 can send a message to a multiple number of designated players, say $p_2, p_3, \dots, p_q$, so that Eve and any other player get no information on the message.

Let V be the set of all players, that is, $V = \{p_1, p_2, \dots, p_n\}$. Let $Q = \{p_1, p_2, \dots, p_q\}$. For a subset Q^* of Q with $|Q^*| = q^* \geq 2$, we define a *graph* $F(Q^*)$, as follows. Let t be a positive integer, and let $T_1, T_2, \dots, T_t$ be t trees such that

- (a) each tree T_i , $1 \leq i \leq t$, contains all the vertices in Q^* and possibly some other vertices in V ;
- (b) any vertex $p_v \in V - Q$ is not contained in at least one of the t trees $T_1, T_2, \dots, T_t$; and
- (c) $T_1, T_2, \dots, T_t$ are pairwise edge-disjoint.

Then $F(Q^*)$ is the union of these t trees $T_1, T_2, \dots, T_t$. We now define a *graph* $\hat{F}$ in terms of graphs $F(Q^*)$. Let s be a positive integer, and let $Q_1, Q_2, \dots, Q_s$ be subsets of Q satisfying the following (a)–(c):

- (a) $Q = Q_1 \cup Q_2 \cup \dots \cup Q_s$;
- (b) $Q_1, Q_2, \dots, Q_s$ are “connecting,” that is, for any indices i and j , $1 \leq i < j \leq s$, there is a sequence $i_1, i_2, \dots, i_k$ of indices such that $i_1 = i$, $i_k = j$, and $Q_{i_\ell} \cap Q_{i_{\ell+1}} \neq \emptyset$ for every ℓ , $1 \leq \ell \leq k - 1$; and

- (c) there exist pairwise edge-disjoint graphs $F(Q_1), F(Q_2), \dots, F(Q_s)$.

Then let $\hat{F}$ be the union of graphs $F(Q_1), F(Q_2), \dots, F(Q_s)$. Clearly $\hat{F}$ is a connected graph, which contains all vertices $p_1, p_2, \dots, p_q$ and possibly some others. It should be noted that $F(Q_i)$ and $F(Q_j)$, $1 \leq i < j \leq s$, may share a common vertex $p_v \notin Q$.

We showed that a player p_1 can send a message m to players $p_2, p_3, \dots, p_q$ absolutely securely if the key sharing graph G contains $\hat{F}$ as a subgraph.

2 Conclusions

For simplicity, we assumed that a message and all secret keys are 1-bit numbers. However, one can easily extend the results to the case where a message and all secret keys are ℓ -bit numbers for any $\ell (\geq 1)$. In this case, $\oplus$ must be a bit-wise exclusive OR.

It is desirable to find a necessary and sufficient condition for the existence of a secure multicast protocol.

References

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Publications

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